A treatise on the principal mathematical instruments employed in surveying, levelling, and astronomy

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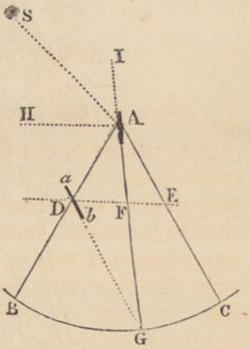
Astronomical instruments.

ASTRONOMICAL INSTRUMENTS.

THE SEXTANT.

It was our intention, before describing tbe Sextant, to devote some space to an account of HADLEY'S Quadrant; but as the construction of both instruments is essentially the same, we shall confine ourselves to a description of the sextant and its uses only, as comprehending the other instrument, and performing with greater correctness all the operations to which the quadrant can be applied. The principle of its construction may be understood from the following demonstration.

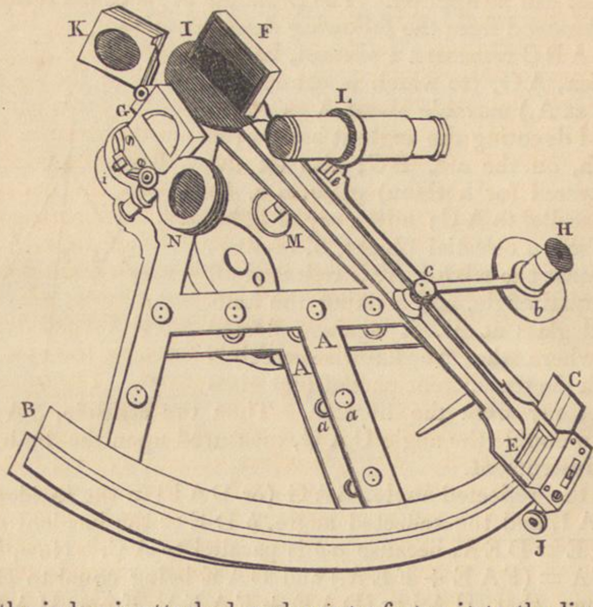


Let A B C represent a sextant, having an index, A G, (to which is attached a mirror at A,) movable about A as a centre, and denoting the angle it has moved through, on the arc, B C; also let the half-silvered (or horizon) glass, *a b*, be fixed parallel to A C; now a ray of light, S, A, from a celestial object, S, impinging against the mirror, A, is reflected off at an equal angle, and striking the half-silvered glass at D, is again reflected to E, where the eye likewise receives through the transparent part of that glass a direct ray from the horizon. Then the altitude, S A H, is equal to double the angle C A G, measured upon the limb, B C, of the instrument.

For the reflected angle, B A G (or D A F) = the incident angle, S A I, and the reflected angle, *b* D E = the incident *a* D A = D A E = D E A, because *a b* is parallel to A C. Now, H A I = D F A = (F A E + F E A,) and D A E being equal to D E A, it follows that H A I = (D A E + F A E.) From H A I and (D A E + F A E) take the equal angles, S A I and D A F, and there remains S A H = 2 F A E, or 2 G A C; or, in other words, the angle of elevation, S A H, is equal to double the angle of inclination of the two mirrors, D G A, being equal to G A C.

Hence the arc on the limb, B C, although only the sixth part of a circle, is divided as if it were 120°, on account of its double being required as the measure of C A B, and it is generally extended to 140°.

The annexed figure represents a sextant of TROUGHTON's construction, having a double frame, A A, connected by pillars, *a a*, &c., thus uniting strength with lightness. The arc, B C, is generally graduated to 10′ of a degree, commencing near the end, C, and it is numbered towards B. The divisions are also continued on the other side of zero, towards C, forming what is called the arc of excess, which is useful in determining the index error of the instrument, as will be explained hereafter. The limb is subdivided by the vernier, E, into 10″, the half of which (or 5″) can be easily estimated: this small quantity is easily distinguishable by the aid of microscope, H, and its reflector, *b*, which are connected by an arm with the index, I E, at the point *c*, round which it turns as a centre, affording the means of examining the whole vernier, the connecting arm being long enough to allow the microscope to pass over the whole length of it.



To the index is attached a clamp to fasten it to the limb, and a tangent screw, J, (in the plate, the clamp is concealed from view,) by which the index may be moved any small quantity, after it is clamped, to render the contact of the objects observed more perfect than can be done by moving it with the hand alone. The upper end, I, terminates in a circle, across which is fixed the silvered index-glass, F, over the centre of motion, and perpendicular to the plane of the instrument. To the frame at G is attached a second glass, called the horizon-glass, the lower half of which only is silvered: this must likewise be perpendicular to the plane of the instrument, and in such a position that its plane shall be parallel to the plane of the index-glass, F, when the vernier is set to 0° (or zero) on the limb, B C. A deviation from this position constitutes the index error before spoken of.

The telescope is carried by a ring, L, attached to a stem, *e*, called the up-and-down piece, which can be raised or lowered by turning the milled screw, M: its use is to place the telescope so that the field of view may be bisected by the line on the horizon-glass that separates the silvered from the unsilvered part. This is important, as it renders the object seen by reflection, and that by direct vision equally bright;* two telescopes and a plain tube, all adapted to the ring, L, are packed with the sextant, one showing the objects erect, and the other inverting them; the last has a greater magnifying power, showing the contact of the images much better. The adjustment for distinct vision is obtained by sliding the tube at the eye-end of the telescope in the inside of the other; this also is the means of adapting the focus to suit different eyes. In the inverting telescope are placed two wires, parallel to each other, and in the middle of the space between them the observations are to be made, the wires being first brought parallel to the plane of the sextant, which may be judged of with sufficient exactness by the eye. When observing with this telescope, it must be borne in mind, that the instrument must be moved in a contrary direction to that which the object appears to take, in order to keep it in the field of view.

Four dark glasses, of different depths of shade and colour, are placed at K, between the index and horizon glasses; also three more at N, any one or more of which can be turned down to moderate the intensity of the light, before reaching the eye, when a very luminous object (as the sun) is observed. The same purpose is effected by fixing a dark glass to the eye-end of the telescope: one or more dark glasses for this purpose generally accompany the instrument. They, however, are chiefly used when the sun's altitude is observed with an artificial horizon, or for ascertaining the index error, as employing the shades attached to the instrument for such purposes, would involve in the result, any error which they might possess. The handle, which is shown at O, is fixed at the back of the instrument. The hole in the middle is for fixing it to a stand, which is useful when an observer is desirous of great steadiness.

* This is not the case when one object is much brighter than the other, as the sun and moon; in taking the distance between which, the screw, M, should be moved more than above stated, until they are both nearly of the same brightness, as an observation can be made better when this is the case than when otherwise.

Of the Adjustments.

The requisite adjustments are the following: the index and horizon glasses must be perpendicular to the plane of the instrument, and their planes parallel to each other when the index division of the vernier is at 0° on the arc, and the optical axis of the telescope must be parallel to the plane of the instrument. We shall speak separately of each of these adjustments.

To examine the Adjustment of the Index-glass.

Move the index forward to about the middle of the limb; then, holding the instrument horizontally with the divided limb from the observer, and the index-glass to the eye, look obliquely down the glass, so as to see the circular arc, by direct view and by reflection, in the glass at the same time; and if they appear as one continued arc of a circle, the index-glass is in adjustment. If it requires correcting, the arc will appear broken where the reflected and direct parts of the limb meet. This in a well-made instrument is seldom the case, unless the sextant has been exposed to rough treatment. As the glass is in the first instance set right by the maker, and firmly fixed in its place, its position is not liable to alter, therefore no direct means are supplied for its adjustment.

To examine the horizon-glass, and set it perpendicular to the Plane of the Sextant.

The position of this glass is known to be right, when by a sweep with the index, the reflected image of any object passes exactly over or covers its image as seen directly; and any error is easily rectified by turning the small screw, *i*, at the lower end of the frame of the glass.

To examine the Parallelism of the Planes of the two Glasses, when the Index is set to Zero.

This is easily ascertained; for, after setting the zero on the index to zero on the limb, if you direct your view to some object, the sun for instance, you will see that the two images (one seen by direct vision through the unsilvered part of the horizon-glass, and the other reflected from the silvered part) coincide or appear as one, if the glasses are correctly parallel to each other; but if the two images do not coincide, the quantity of their deviation constitutes what is called the index error. The effect of this error on an angle measured by the instrument is exactly equal to the error itself: therefore, in modern instruments, there are seldom any means applied for its correction, it being considered preferable to determine its amount previous to observing, or immediately after, and apply it with its proper sign to each observation. The amount of the index error may be found in the following manner: clamp the index at about 30 minutes to the left of zero, and looking towards the sun, the two images will appear either nearly in contact or overlapping each other; then perfect the contact, by moving the tangent-screw, and call the minutes and seconds denoted by the vernier, the reading on the arc. Next place the index about the same quantity to the right of zero, or on the arc of excess, and make the contact of the two images perfect as before, and call the minutes and seconds on the arc of excess* the reading off the arc; and half the difference of these numbers is the index error; additive when the reading on the arc of excess is greater than that on the limb, and subtractive when the contrary is the case.

EXAMPLE.

		′	″
Reading on the arc . . .		31	56
,, off the arc . . .		31	22
Difference		0	34
Index error . . .	= —	0	17

In this case the reading on the arc being greater than that on the arc of excess, the index error, =17 seconds, must be subtracted from all observations taken with the instrument, until it be found, by a similar process, that the index error has altered. One observation on each side of zero is seldom considered enough to give the index error with sufficient exactness for particular purposes: it is usual to take several measures each way; "and half the difference of their means will give a result more to be depended on than one deduced from a single observation only on each side of zero." A proof of the correctness of observations for index error is obtained by adding the above numbers together, and taking one-fourth of their sum, which should be equal to the sun's semidiameter, as given in the Nautical Almanac. When the sun's altitude is low, not exceeding 20° or 30°, his horizontal instead of his perpendicular diameter should be measured, (if the observer intends to compare with the Nautical Almanac, otherwise there is no necessity;) because the refraction at such an altitude affects the lower border (or limb) more than the upper,

* When reading off the arc of excess, the vernier must be read backwards, or from its contrary end, as explained at page 7.

so as to make his perpendicular diameter appear less than his horizontal one, which is that given in the Nautical Almanac: in this case the sextant must be held horizontally.

To make the Line of Collimation of the Telescope parallel to the Plane of the Sextant.

This is known to be correct, when the sun and moon, having a distance of 90° or more, are brought into contact just at the wire of the telescope which is nearest the plane of the sextant, fixing the index, and altering the position of the instrument to make the objects appear on the other wire; if the contact still remains perfect, the axis of the telescope is in proper adjustment; if not, it must be altered by moving the two screws which fasten, to the up-and-down piece, the collar into which the telescope screws. This adjustment is not very liable to be deranged.

Having now gone through the principle and construction of the sextant, it remains to give some instructions as to the manner of using it.

It is evident that the plane of the instrument must be held in the plane of the two objects, the angular distance of which is required: in a vertical plane, therefore, when altitudes are measured; in a horizontal or oblique plane, when horizontal or oblique angles are to be taken. As this adjustment of the plane of the instrument is rather difficult and troublesome to the beginner, he need not be surprised nor discouraged, although his first attempts may not answer his expectations. The sextant must be held in the right hand, and as slack as is consistent with its safety, for in grasping it too hard the hand is apt to be rendered unsteady.

When the altitude of an object, the sun, for instance, is to be observed, the observer, having the sea horizon before him, must turn down one or more of the dark glasses, or shades, according to the brilliancy of the object; and directing his sight to that part of the horizon immediately beneath the sun, and holding the instrument vertically, he must with the left hand lightly slide the index forward, until the image of the sun, reflected from the index glass, appears in contact with the horizon, seen through the unsilvered part of the horizon glass. Then clamp it firm, and gently turn the tangent-screw, to make the contact of the upper or lower limb of the sun and the horizon perfect, when it will appear a tangent to his circular disc.* If an artificial

* If the observer knows his latitude approximately, he may find the meridional altitude nearly, to which he may previously set his instrument; when he will not only find his object more easily, but have only a small quantity to move the index to perfect the observation.

Take from the Nautical Almanac the declination of the object, and if it be of the same name with the latitude, add it to the co-latitude; if of a different name, subtract it: the sum or difference will be the meridian altitude.

horizon is employed, the two images of the sun must be brought into contact with each other; but this will be explained when speaking of that instrument. To the angle read off apply the index error, and then add or subtract the sun's semidiameter, as given in the Nautical Almanac, according as the lower or upper limb is observed, to obtain the apparent altitude of the sun's centre. Before we can use this observation for determining the time, the latitude, &c., it must be further corrected for refraction and parallax, to obtain the true altitude, subtracting the former and adding the latter; and when the sea horizon is employed, a quantity must also be subtracted for the dip, which is unnecessary when the altitude is taken by means of an artificial horizon.

Tables for obtaining the above corrections may be found in Mr. Baily's Astronomical Tables, &c., in the Requisite Tables, or in any modern work on navigation.

EXAMPLE.

			°	′	″
Obs. alt. of the sun's lower limb		=	61	13	5
Index error		=	—		17
Apparent altitude		=	61	12	48,0
* { Sun's semidiameter		=	+	15	46,9
{ ,, parallax		=	+	0	4,0
Refraction..................	″ — 34,4		61	28	38,9
Dip of the horizon, for an elevation of 18 feet...	—4 3,0	=	—	4	37,4
True altitude of the sun's centre		=	61	24	1,5

If the observer is ignorant of the precise moment of the object's being on the meridian, he should, by a slow and gradual motion of the tangent-screw, keep the observed limb in contact with the horizon as long as it continues to rise; and immediately on the altitudes appearing to diminish, cease from observing, and the angle then read on the instrument will be the meridian altitude.

After what has been advanced, little need be said about observing lunar distances, whether of the moon and the sun, or the moon and a fixed star or planet, except that the instrument must be held in the plane of the two objects, and it is generally preferable to direct the telescope to the fainter object, particularly if a star, as it can be more easily kept in view when seen directly

* An observation of a star requires no correction for either parallax or semidiameter.

than it can when seen by reflection. If the brighter object is to the left, the sextant must be held with the face downwards.

The enlightened limb of the moon is always to be brought into contact with the sun or star, even though the moon's image is made to pass beyond the sun or star before the desired contact can be obtained.

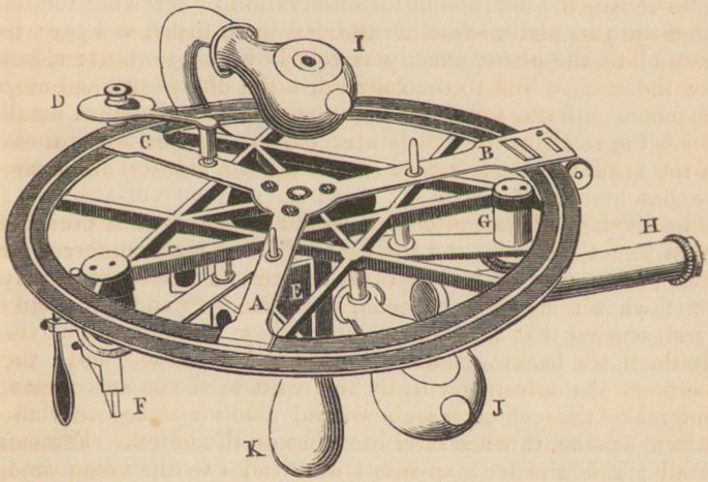
Perhaps the best method of taking a lunar distance is, not to attempt to make the contact perfect by the tangent-screw, but when the nearest limbs are observed, make the objects overlap each other a little when they are receding, or leave a small space between them when they are approaching, and wait till the contact is perfect, and the reverse, when the furthest limbs are observed.

The altitudes of the two objects should be observed at the same instant as the distance, and the time noted by a chronometer, or watch: this would require several observers; but one person may take them all, by having recourse to the following method: "First, observe the altitude of the sun or star; secondly, the altitude of the moon; then any number of distances; next the altitude of the moon, and lastly the altitude of the sun or star, noting the times of each by a watch. Now add together the distances and times when they were observed, and take the mean of each; and in order to reduce the altitudes to the mean time, making the following proportion: As the difference of times between the observations is to the difference of their altitudes, so is the difference between the time that the first altitude was taken and the mean of the times at which the distances were observed, to a fourth number: which, added to or subtracted from the first altitude, according as it is increasing or decreasing, will give the altitude reduced to the mean time."

The angular distances of terrestrial objects are measured by the sextant in the same manner as those of celestial ones; but if the objects are not in the same horizontal plane, a reflecting instrument will not give their horizontal angular distance. But this may be obtained nearly by measuring their angular distances from an object in or near the horizon, which subtends a great angle with both, and the sum, or the difference of the angles so measured, will be nearly the required horizontal angle.

Of the sextant, it has been said, that it is in itself a portable observatory; and it is doubtless one of the most generally useful instruments that has ever been contrived, being capable of furnishing data to a considerable degree of accuracy for the solution of a numerous class of the most useful astronomical problems; affording the means of determining the time, the latitude and longitude of a place, &c., for which, and many other purposes, it is invaluable to the land surveyor as well as the navigator.

TROUGHTON'S REFLECTING CIRCLE.



The above figure represents this instrument, which in principle and use is the same as the sextant. It has three vernier readings, A B C, moving round the same centre as the index-glass, E, which is upon the opposite face of the instrument. One of the verniers, B, carries the clamp and tangent-screw. D, represents the microscope for reading the verniers; it is similar to the one used in reading the sextant, and is adapted to each index-bar, by slipping it on a pin placed for that purpose, as shown in the figure. The horizon-glass is shown at F. The barrel, G, contains the screws for giving the up-and-down motion to the telescope; it is put in action by turning the milled-head under the barrel. H is the telescope, adapted to the instrument in a manner similar to that of the sextant. I and J are two handles fixed parallel to the plane of the circle, and a third handle, K, is screwed on at right angles to that plane, and can be transferred to the opposite face of the instrument by screwing it into the handle, I; the use of this extra handle is for convenience in reading and in holding the instrument, when observing angles that are nearly horizontal; it can be shifted, according as the face of the instrument is held upwards or downwards. The requisite dark glasses are attached to the frame-work of the circle, to be used in the same manner and for the same purposes as those of the sextant. With respect to the adjustments and application of this instrument, we cannot do better than use the words of the inventor, Mr. TROUGHTON, contained in a paper which he calls

"*Directions for observing with Troughton's Reflecting Circle.*

"Prepare the instrument for observation by screwing the telescope into its place, adjusting the drawer to focus, and the wires parallel to the plane, exactly as you do with a sextant: also set the index forwards to the rough distance of the sun and moon, or moon and star; and holding the circle by the short handle, direct the telescope to the fainter object, and make the contact in the usual way. Now read off the degree, minute, and second, by that branch of the index to which the tangent-screw is attached; also, the minute and second shown by the other two branches; these give the distance taken on three different sextants; but as yet, it is only to be considered as half an observation: what remains to be done, is to complete the whole circle, by measuring that angle on the other three sextants. Therefore set the index backwards nearly to the same distance, and reverse the plane of the instrument, by holding it by the opposite handle, and make the contact as above, and read off as before what is shown on the three several branches of the index. The mean of all six is the true apparent distance, corresponding to the mean of the two times at which the observations were made.

"When the objects are seen very distinctly, so that no doubt whatever remains about the contact in both sights being perfect, the above may safely be relied on as a complete set; but if, from the haziness of the air, too much motion, or any other cause, the observations have been rendered doubtful, it will be advisable to make more: and if, at such times, so many readings should be deemed troublesome, six observations, and six readings may be conducted in the manner following: Take three successive sights forwards, exactly as is done with a sextant; only take care to read them off on different branches of the index: also make three observations backwards, using the same caution: a mean of these will be the distance required. When the number of sights taken forwards and backwards are unequal, a mean between the means of these taken backwards and those taken forwards will be the true angle.

"It need hardly be mentioned, that the shades, or dark glasses, apply like those of a sextant, for making the objects nearly of the same brightness; but it must be insisted on, that the telescope should, on every occasion, be raised or lowered, by its proper screw, for making them perfectly so.

"The foregoing instructions for taking distances, apply equally for taking altitudes by the sea or artificial horizon, they being no more than distances taken in a vertical plane. Meridian altitudes cannot, however, be taken both backwards and forwards the same day, because there is not time: all therefore that can be done, is, to observe the altitude one way, and use the index error; but even here, you have a mean of that altitude, and this error, taken on three different sextants. Both at sea and land, where the observer is stationary, the meridian altitude should be observed forwards one day, and backwards the next, and so on alternately from day to day; the mean of latitudes, deduced severally from such observations, will be the true latitude; but in these there should be no application of index error, for that being constant, the result would in some measure be vitiated thereby.

"When both the reflected and direct images require to be darkened, as is the case when the sun's diameter is measured and when his altitude is taken with an artificial horizon, the attached dark glasses ought not to be used: instead of them, those which apply to the eye-end of the telescope will answer much better: the former having their errors magnified by the power of the telescope, will, in proportion to this power, and those errors, be less distinct than the latter.

"In taking distances, when the position does not vary from the vertical above thirty or forty degrees, the handles which are attached to the circle are generally most conveniently used; but in those which incline more to the horizontal, that handle which screws into a cock on one side, and into the crooked handle on the other, will be found more applicable.

"When the crooked handle happens to be in the way of reading one of the branches of the index, it must be removed, for the time, by taking out the finger-screw, which fastens it to the body of the circle.

"If it should happen that two of the readings agree with each other very well, and the third differs from them, the discordant one must not on any account be omitted, but a fair mean must always be taken.

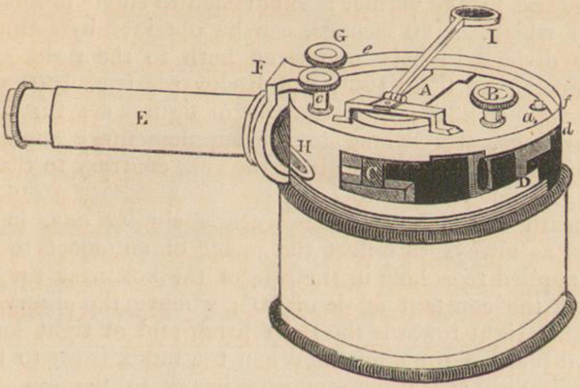
"It should be stated, that when the angle is about thirty degrees, neither the distance of the sun and moon, nor an altitude of the sun, with the sea horizon, can be taken backwards; because the dark glasses at that angle prevent the reflected rays of light from falling on the index-glass; whence it becomes necessary, when the angle to be taken is quite unknown, to observe forwards first, where the whole range is without interruption; whereas in that backwards, you will lose sight of the reflected image about that angle. But in such distances, where the sun is out of the question, and when his altitude is taken with an artificial horizon, (the shade being applied to the end of the telescope,) that angle may be measured nearly as well as any other; for the rays incident on the index-glass will pass through the transparent half of the horizon-glass, without much diminution of their brightness.

" The advantages of this instrument, when compared with the sextant, are chiefly these : the observations for finding the index error are rendered useless, all knowledge of that being put out of the question, by observing both forwards and backwards. By the same means the errors of the dark glasses are also corrected ; for, if they increase the angle one way, they must diminish it the other way by the same quantity. This also perfectly corrects the errors of the horizon-glass, and those of the index-glass very nearly. But what is still of more consequence, the error of the centre is perfectly corrected by reading the three branches of the index ; while this property combined with that of observing both ways, probably reduces the errors of dividing to one-sixth part of their simple value. Moreover, angles may be measured as far as one hundred and fifty degrees, consequently the sun's double altitude may be observed when his distance from the zenith is not less than fifteen degrees ; at which altitude, the head of the observer begins to intercept the rays of light incident on the artificial horizon ; and, of course, if a greater angle could be measured, it would be of no use in this respect.

" This instrument, in common with the sextant, requires three adjustments. First, the index-glass perpendicular to the plane of the circle. This being done by the maker, and not liable to alter, has no direct means applied to the purpose ; it is known to be right, when, by looking into the index-glass, you see that part of the limb which is next you, reflected in contact with the opposite side of the limb, as one continued arc of a circle : on the contrary, when the arc appears broken, where the reflected and direct parts of the limb meet, it is a proof that it wants to be rectified. The second is, to make the horizon-glass perpendicular. This is performed by a capstan-screw, at the lower end of the frame of that glass ; and is known to be right, when, by a sweep of the index, the reflected image of any object will pass exactly over, or cover the image of that object seen directly. The third adjustment is, for making the line of collimation parallel to the plane of the circle. This is performed by two small screws, which also fasten the collar into which the telescope screws to the upright stem on which it is mounted ; this is known to be right, when the sun and moon, having a distance of one hundred and thirty degrees, or more, their limbs are brought in contact, just at the outside of that wire which is next to the circle ; and then, examining if it be the same, just at the outside of the other wire : its being so is the proof of adjustment.

" Should these hints about the adjustments set any over-handy gentleman on tormenting his instrument, it will not be what was intended by them ; they were added, that, in case of accident, those who are so unfortunate, might be enabled thereby to put their own instrument in order."

THE BOX SEXTANT.



This useful little instrument, which is represented in the above figure, might, perhaps with more propriety, have been classed as a surveying instrument, it being chiefly used in that business. The principle of its construction and adjustments is precisely the same as the sextant before described; a minute description, therefore, would be little more than a recapitulation of what has already been advanced. A is the index, which instead of being moved along the divided limb, *e f*, by the hand, has a motion given to it by a rack and pinion, concealed within the box, and turned by the milled head B, which acts as the tangent-screw does to the index of the large sextant. The glasses (shown at C and D) are within the box, by which they are protected from injury, and their adjustments, when once perfected, kept secure; so much so, that it would require considerable violence to derange them. The horizon-glass, D, alone has a contrivance for adjustment at *a* and *d*, both to set it perpendicular to the plane of the instrument, and to correct or reduce the index error, which, in this instrument, had better be kept correct, as it is not so likely to get out of order as in the large sextant, which, as we have before observed, seldom admits of its index error being rectified. The key, *c*, is formed to fit both squares at *a* and *d*, to make the adjustments, and it is generally tapt into some spare place in the instrument, as at *c*, that it may be always safe and at hand.

It is supplied with a telescope, E, which screws into a shoulder-piece, F, and can be attached to the box by the screw, G: this can be applied or not, at the pleasure of the observer, as there is a contrivance at H to enable him to observe without the telescope, if he prefers plain sights. Two dark glasses are placed within the box, and there is also one adapted to the eye-end of the telescope.

The angle is read off by the help of the glass, I, which being mounted with a joint, can be moved over the vernier on any part of the limb. The instrument is divided to 30 minutes of a degree, and by the vernier is subdivided to single minutes, one-half of which, or 30 seconds, can be obtained by estimation.

The divided limb is numbered both to the right and left, commencing at 0° to 120°, and backwards from 120° to 180°, and beyond to 230°; the latter row of figures are furthest from the divisions, and belong to the supplementary angles; their zero division of the vernier is at the end, contrary to that of the angles, reading from 0° to 120°.

Beneath the index-glass is fixed a similar one, in such a manner as always to reflect the image of an object to the eye when applied to a hole in the side of the box near the division 120°, at the constant angle of 90°; whence the observer must direct his sight towards the right hand, and at right angles, to the real place of the object. When the index is set to 180°, its glass will also reflect an opposite image to the eye at right angles to the left hand, (the two glasses then being exactly across each other;) consequently an eye looking through the hole near the division 120° will (if the adjustments be perfect) perceive objects 180° apart to coincide, at right angles to a line connecting them. Thus a point can be found in line between two stations: the observer, with the instrument set as above, having placed himself as nearly in the line as he can guess, must apply his eye to the hole near 120°, and looking at right angles to his station line, step backwards or forwards, until he perceives the two distant objects to coincide, when the spot he stands on will be a point in the line joining the objects: to verify this, he should then turn himself half round, and looking in the opposite direction, see if the two objects still coincide, which they will do, if the adjustments of the instrument are correct. If they do not appear in junction, move as before, until you find the spot where they do; then, half way between the two spots so found, will be the true point on the line required.

"The adjustment of this part, as well as the method of observing supplemental angles with it, is performed thus: Choose two objects in the horizon, the further apart the better, but not nearer than 140°; turn your face at right angles to the right-hand object, so as to get sight of its image in the fixed glass; then, by moving the index, bring the image of the other object, seen in the index-glass, exactly to coincide with it on the line of separation of the two glasses: read off the angle, turn yourself half round, and take in like manner the angle which the same objects make the other way. It is evident that the sum of the two angles should be 360°, and also, that if they exceed that quantity, half the excess must be subtracted, and if they fall short of it, half the defect must be added, to obtain the true angle. It is, perhaps, better to allow for the errors than to adjust them; but the latter may be done by applying the key, *c*, to a square underneath the box."

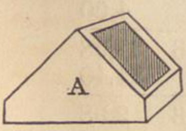
The lid of the box is contrived to screw on the bottom, (as is shown in the plate,) where it makes a convenient handle for holding the instrument.

Since writing the above, we have been shown by Mr. Macneill, an excellent contrivance of his, for taking altitudes or depressions with the box-sextant, which consists of two small spirit-levels fixed at the back of the horizon-glass, at right angles to each other, so that standing before the object, you look perpendicularly down through the plane-sight, and moving the index bring the image of the object to appear with the levels, which must have their air-bubbles in the centre of their tubes. The reading of the instrument will then show the supplement of the zenith distance, and its complement to 90° will be the angle required; elevated if more than 90°, and depressed if less than 90°.

THE ARTIFICIAL HORIZON.

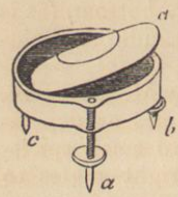
When the altitude of a celestial object is to be taken at sea, the observer has the natural (or sea) horizon, as a line of departure; but on shore, he is obliged to have recourse to an artificial one, to which his observation may be referred: this consists of a reflecting plane parallel to the natural horizon, on which the rays of the sun or other object falling, are reflected back to an eye placed in a proper position to receive them; the angle between the real object and its reflected image, being then measured with the sextant, is double the altitude of the object above the horizontal plane.

Various natural as well as artificial reflecting surfaces have been made by mechanical arrangements, to afford the means of obtaining double angles: such as pouring water, oil, treacle, or other fluid substances into a shallow vessel; and to prevent the wind giving a tremulous motion to its surface, a piece of thin gauze, talc, or plate-glass, whose surfaces are perfectly plane and parallel, may be placed over it, when used for observation. But the most accurate kind of artificial horizon is that in which fluid quicksilver forms the reflecting surface, the containing vessel being placed on a solid basis, and protected from the influence of the wind. The adjoining figure represents an instrument of this kind.



The mercury is contained in an oblong wooden trough, placed under the roof A, in which are fixed two plates of glass whose surfaces are plane and parallel to each other. This roof effectually screens the surface of the metal from being agitated by the wind, and when it has its position reversed at a second observation, any error occasioned by undue refraction at either plate of glass will be corrected.

Another and more portable contrivance for an artificial horizon, is represented in the following figure, which consists of a



circular plate of black glass about two inches diameter, mounted on a brass stand, half an inch deep, with three foot-screws, *a b c*, to set the plane horizontal; the horizontality being determined thus by the aid of a short spirit-level, *d*, having under the tube a face ground plane on which it lies in contact with the reflecting surface; place the level on the glass in a direction parallel to the line joining two of the three foot-screws, as *a* and *b*, then move one of these screws till the bubble remains in the middle of the tube in both the reversed positions of the level, and the plate will be horizontal in that direction; then place the level at right angles to its former position, and turn the third foot-screw back or forwards till the bubble again settles in the middle of its tube, the former levelling remaining undisturbed, and the plane will then be horizontal. This instrument, from its portability, is extremely convenient for travellers, as when packed in its case it can be carried in the pocket without being any incumbrance.

When an artificial horizon is used, the observer must place himself at such a distance that he may see the reflected object as well as the real one; then having the sextant properly adjusted, the upper or lower limb of the sun's image (supposing that the object) reflected from the index-glass, must be brought into contact with the opposite limb of the image reflected from the artificial horizon, observing that when the inverting telescope is used, the upper limb will appear as the lower, and *vice versâ*;* the angle shown on the instrument, when corrected for the index error, will be double the altitude of the sun's limb above the horizontal plane; to the half of which, if the semidiameter, refraction, and parallax be applied, the result will be the true altitude of the centre.

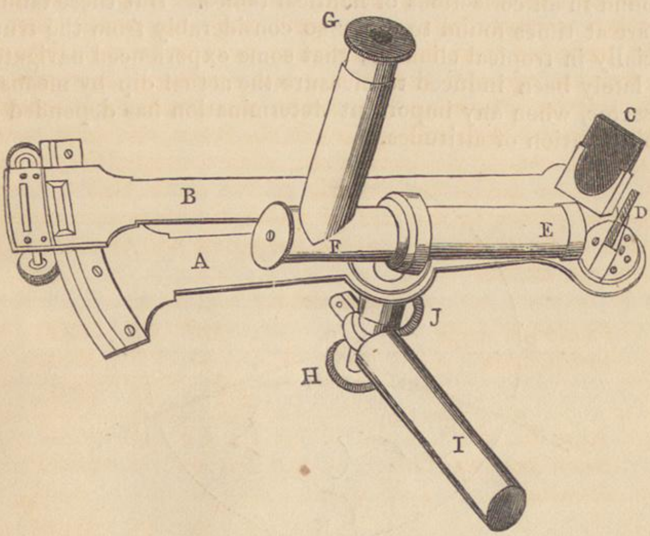
EXAMPLE.

		°	′	″
Observed angle		122	25	50,00
Index error		—		17,05
	2)	122	25	32,95
App. alt.		61	12	46,47
Semidiameter		+	15	46,91
Parallax		+		4,00
		61	28	37,38
Refraction		—		34,40
True alt. of sun's centre . .		61	28	2,98

* When the contact is formed at the lower limb, the images will separate shortly after the contact has been made, if the altitude be increasing; but if the altitude be decreasing, they will begin to overlap; but when the contact is formed at the upper limb, the reverse takes place. An observer, if in doubt as to which limb he has been observing, should watch the object for a short time after he has made the observation.

THE DIP-SECTOR.

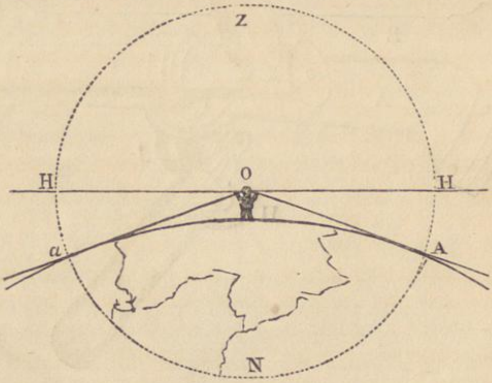
When the late Professor VINCE was engaged in making observations upon extraordinary refraction at Ramsgate, Mr. TROUGHTON contrived and constructed for his use an instrument which he called a Refraction-Sector. About five years afterwards, when preparations were making for the first of the late North Polar Expeditions, Mr. TROUGHTON was applied to by the late Dr. WOOLLASTON, to make him an instrument on the principle of the back observation with the quadrant, to send with the expedition, to measure the dip of the horizon; but upon Mr. TROUGHTON's producing his Refraction-Sector, which was as well adapted to Dr. WOOLLASTON's purpose as that for which it was devised, the Doctor immediately ordered one to be made for him, and named it a Dip-Sector; proposing at the same time an improvement in the construction of the handle, which, on his suggestion, was made to turn on a centre, to be placed in any position, for convenience in use, or packing in its case; that made for Mr. VINCE having two fixed handles, at right angles to the face of the instrument.



The preceding figure represents this instrument: A is the sector, B the index, with its clamp and tangent-screw, exactly similar to that of the sextant: the index-glass, C, and the horizon-glass, D, are fixed at right angles to the plane of the instrument. The telescope, E F, is fitted into a collar, having an up-and-down motion given to it by turning the screw H; the two images of the horizon can thus be made to appear of the shades most favourable for observation. G represents the eye-piece fixed at right angles to the telescope, and a diagonal mirror is placed in the telescope at F, to change the direction of the rays of light, from E F, to F G, in which the observer looks.

The handle, I, turns upon a centre, and is held firmly in any position by tightening the clamp-screw, J. In use it is fixed perpendicular to the length of the instrument, and when wanted it can be turned half round, and fixed in a similar position on the other side, a position in which it is required to be when the instrument is reversed for the second observation; it is turned under and parallel to the instrument when packed in its case.

The dip of the horizon, which varies with the height of the observer above the surface of the earth, may always be computed when the height is known; but as a correction of altitudes observed from the horizon of the sea, it is combined with the effects of refraction upon the apparent place of the horizon, which appears elevated above its true place; and as the effects of refraction are extremely variable, the dip obtained by computation is necessarily very uncertain. Tables containing the dip for various altitudes, allowing for the mean effect of refraction, are to be found in all collections of nautical tables. But these tabular dips are at times found to differ so considerably from the truth, especially in tropical climates, that some experienced navigators have lately been induced to measure the actual dip by means of the sector, when any important determination has depended on the observation of altitudes.

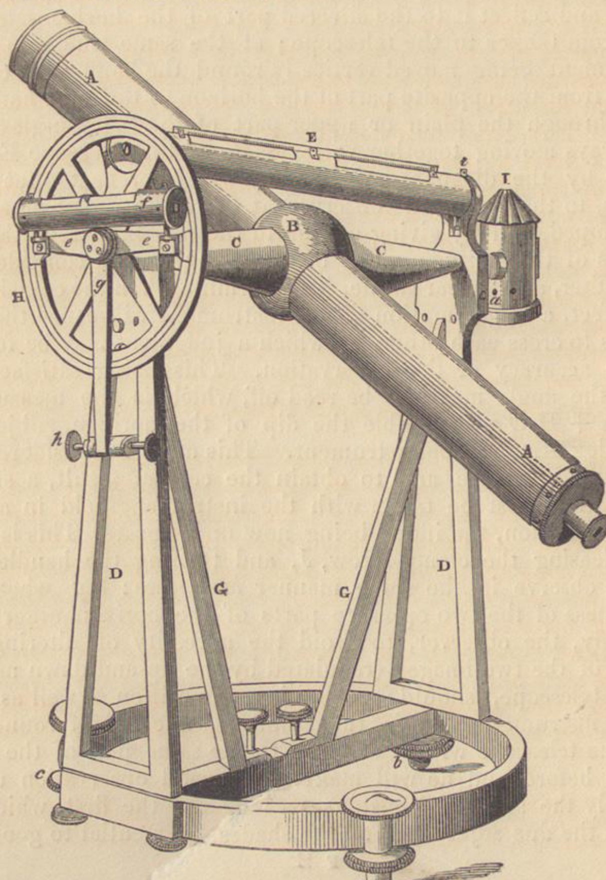


In the above diagram, A *a* represents a portion of the earth's surface, and O the place of an observer; H O H will be his true horizon, O A and O *a* his visible horizon; these rays being tangents to the earth's surface at A and *a*; the angle, H O A, or H O *a*, is the dip of the horizon, which it is the business of the dip-sector to measure. But the arcs to be measured by this instrument, for the purpose of obtaining the dip, are A Z *a* and A N *a*, the former of which is 180° + double the dip, and the latter 180° — double the dip, therefore the fourth part of the difference is the measure of the dip. But as the instrument is constructed, only double the dip affected by index error is read from it, and the index error is made so great that the readings are both on the same side of zero, therefore the fourth part of the difference of the readings is the dip angle.

In observing, the face of the instrument must be held in a vertical plane, and lengthwise, in a line with the opposite parts of the horizon whose dip is required; the eye-tube, G F, (page 65) will then be horizontal, and the observer will be looking at right angles to those points of the horizon which he wishes to observe. Suppose the instrument to be held as represented in our engraving, with the index uppermost, the observer will be looking in the direction, G F, when by giving motion to the index B, its glass C will receive *a ray* from the visible horizon on the left hand, and reflect it to the silvered part of the horizon-glass D, and from thence to the telescope; at the same time the whole instrument being moved vertically round the hand as a centre, *a ray* from the opposite part of the horizon to the right hand will pass through the plain or upper part of the horizon-glass, and both rays moving together will pass down the telescope E to F, where by the diagonal mirror they will be reflected, at right angles, to the eye of the observer at G. The index must now be clamped, and by giving motion to the tangent-screw, the two images of the horizon must be made exactly to coincide with each other, and appear as one. To determine when the coincidence is perfect, a slight motion of the instrument will cause the two images to cross each other, by which a judgment may be formed of the accuracy of the observation. This being satisfactorily done, the angle may now be read off, which is the measure of H O A + H O *a*, or double the dip of the horizon, subject to the index error of the instrument. This must be considered but half an observation, and to obtain the correct result, a second observation must be taken with the instrument held in an inverted position, the index being now undermost. This is done by releasing the clamp-screw, J, and turning the handle half round, observe in the same manner as before: but when the brightness of the two opposite parts of the horizon differ considerably, the observer, to avoid the necessity of altering the shades of the two images, (regulated by the up-and-down motion of the telescope,) should reverse his own position as well as that of his instrument, that is, turn himself exactly half round, for then the telescope will be directed to the same part of the horizon as before, and he will make the second observation under precisely the same circumstances as he did the first, which, as well as the due adjustment of the shades, is essential to good observing. The reading of the second observation will also give double the dip, affected by the same index error as before; and as both readings are on the same side of zero, *one fourth* of their difference will give the true result. Several observations should be taken in each position of the instrument, and the mean taken as the final result.

"In using this instrument at sea for the first time, considerable difficulty arises from the constant change in the plane of the instrument, from the perpendicular position in which it is absolutely necessary that it should be held, in order to obtain a correct observation. What at first appears to be a defect, however, is a real advantage, namely, that whenever it is held in the least degree out of the vertical plane, the two horizons (that seen direct and the reflected one) cross each other, and it is only when the plane is vertical that the horizons can appear parallel."

THE PORTABLE TRANSIT-INSTRUMENT.



The Transit is a meridional instrument employed, in conjunction with a clock or chronometer, for observing the passage of celestial objects across the meridian, either for obtaining correct time, or determining their difference of right ascension; the latter of which, in the case of the moon and certain stars near her path, that differ but little from her in right ascension, affords the best means of determining the difference of longitude between any two places where corresponding observations may have been made. Such being more especially the use of the *portable* transit instrument, it forms a valuable accession to the apparatus of the scientific traveller, who remaining a short time at any station, is enabled thereby to adjust his time-keepers, both with ease and accuracy, and to obtain the best data for finding his longitude. It also may be employed very successfully in determining the latitude.*

The preceding figure represents this instrument as constructed by Mr. TROUGHTON, when the telescope does not exceed twenty inches, or two feet focal length. The telescope-tube, A A, is in two parts, and connected together by a sphere, B, which also receives the larger ends of two cones, C C, placed at right angles to the direction of the telescope, and forming the horizontal axis. This axis terminates in two cylindrical pivots, which rest in Y's fixed at the upper end of the vertical standards, D D. One of the Y's possesses a small motion in azimuth, communicated by turning the screw, *a*; in these Y's the telescope turns upon its pivots. But, that it may move in a vertical circle, the pivots must be precisely on a level with each other, otherwise the telescope will revolve in a plane oblique (instead of perpendicular) to the horizon. The levelling of the axis, as it is called, is therefore one of the most important adjustments of the instrument, and is effected by the aid of a spirit-level, E, which is made for this purpose to stride across the telescope, and rest on the two pivots.

The standards, D D, are fixed by screws upon a brass circle, F, which rests on three screws, *b c d*, forming the feet of the instrument, by the motion of which the operation of levelling is performed. The two oblique braces, G G, are for the purpose of steadying the supports, it being essential for the telescope to have not only a free but a steady motion. On the extremity of one of the pivots, which extends beyond its Y, is fixed a circle, H, which turns with the axis while the double vernier, *e e*, remains stationary in a horizontal position, and shows the altitude to which the telescope is elevated. The verniers are set horizontal by means of a spirit-level, *f*, which is attached to them,

* The transit-instrument is also now much employed by the most eminent civil engineers, in setting out the lines of direction, and the working shafts, in tunnelling; which it is capable of doing with the greatest precision.

and they are fixed in their position by an arm of brass, *g*, clamped to the supports by a screw at *h*: the whole of this apparatus is movable with the telescope, and when the axis is reversed, can be attached in the same manner to the opposite standard.

Near the eye-end, and in the principal focus of the telescope, is placed the diaphragm, or wire-plate, which in the theodolite or levelling telescope need only carry two cross wires, but in this instrument it has five vertical and two horizontal wires. The centre vertical wire ought to be fixed in the optical axis of the telescope, and perpendicular with respect to the pivots of the axis. It will be evident upon consideration, that these wires are rendered visible in the daytime by the rays of light passing down the telescope to the eye; but at night, except when a very luminous object, as the moon, is observed, they cannot be seen. Their illumination is therefore effected by piercing one of the pivots, and admitting the light of a lamp fixed on the top of one of the standards, as shown at I; which light is directed to the wires by a reflector placed diagonally in the sphere B; the reflector having a large hole in its centre, does not interfere with the rays passing down the telescope from the object, and thus the observer sees distinctly both the wires and the object at the same time: when however the object is very faint, (as a small star,) the light from the lamp would overpower its feeble rays: to remedy this inconvenience, the lamp is so constructed, that by turning a screw at its back, or inclining the opening of the lantern, more or less light may be admitted to the telescope, to suit the circumstances of the case.

The telescope is furnished with a diagonal eye-piece, by which stars near the zenith may be observed without inconvenience.

Of the Adjustments.

Upon setting the instrument up, it should be so placed that the telescope, when turned down to the horizon, should point north and south as near as can possibly be ascertained. This of course can be but approximate, as the correct determination of the meridian can only be obtained by observation, after the other adjustments are completed.

The first adjustment is that of the line of collimation. Direct the telescope to some small, distant, well-defined object, (the more distant the better,) and bisect it with the middle of the central vertical wire; then lift the telescope very carefully out of its angular bearings, or Y's, and replace it with the axis reversed; point the telescope again to the same object, and if it be still bisected, the collimation adjustment is correct; if not, move the wires one half the error, by turning the small screws which hold the diaphragm near the eye-end of the telescope, and the adjustment will be accomplished; but, as half the deviation may not be correctly estimated in moving the wires, it becomes necessary to verify the adjustment by moving the telescope the other half, which is done by turning the screw *a;* this gives the small azimuthal motion to the Y before spoken of, and consequently to the pivot of the axis which it carries. Having thus again bisected the object, reverse the axis as before, and if half the error was correctly estimated, the object will be bisected upon the telescope being directed to it; if not quite correct, the operation of reversing and correcting half the error, in the same manner, must be gone through again, until, by successive approximations, the object is found to be bisected in both positions of the axis; the adjustment will then be perfect. The collimation adjustment may likewise be examined from time to time, by observing the transit of Polaris, or any other close circumpolar star, over the first three wires, which gives the intervals in time from the first to the second, and from the first to the third wire; and then reversing the axis, observe the same intervals in a reverse order, as the wires which were the three first, in the former position, will now be the three last: if the intervals in the first observations are exactly the same as the intervals in the second, the collimation adjustment is correct; but should the corresponding intervals differ, such difference points out the existence of an error, which must be removed, as before described, one half by the collimating screws, and the other half by the azimuthal motion of the instrument.

It is desirable that the central, or middle wire (as it is usually termed) should be truly vertical; as we should then have the power of observing the transit of a star on any part of it, as well as the centre. It may be ascertained whether it is so, by elevating and depressing the telescope: when directed to a distant object, if it is bisected by every part of the wire, the wire is vertical; if otherwise, it should be adjusted, by turning the inner tube carrying the wireplate, until the above test of its verticality be obtained, or else care must be taken that the observations are made near the centre only; the other vertical wires are placed by the maker equidistant from each other and parallel to the middle one, therefore, when the middle one is adjusted, the others are so too; he also places the two transverse wires at right angles to the vertical middle wire. These adjustments are always performed by the maker, and but little liable to derangement. When, however, they happen to get out of order, and the observer wishes to correct them, it is done by loosening the screws which hold the eye-end of the telescope in its place, and turning the end round a small quantity by the hand until the error is removed. But this operation requires very delicate handling, as it is liable to remove the wires from the focus of the object-glass.

The axis on which the telescope turns must next be set horizontal: to do this, apply the level to the pivots, bring the air-bubble to the centre of the glass-tube, by turning the foot-screw, *b*, which raises or lowers that end of the axis, and consequently the level resting upon it; then reverse the level by turning it end for end, and if the air-bubble still remains central, the axis will be horizontal, but if not, half the deviation must be corrected by the foot-screw, *b*, and the other half by turning the small screw, *i*, at one end of the level, which raises or lowers the glass-tube (containing the air-bubble) with respect to its supports, which rest upon the pivots. This, like most other adjustments, frequently requires several repetitions before it is accomplished, on account of the difficulty of estimating exactly half the error.

Having set the axis on which the telescope turns, parallel to the horizon, and proved the correct position of the central wire or line of collimation, making it describe a great circle perpendicular to that axis, it remains finally to make it move in that vertical circle which is the meridian.

We have supposed the instrument to be nearly in the meridian, the next step is to determine the amount of its deviation, and then by successive approximations to bring it exactly into that plane: one of the methods of accomplishing this, is to observe the time of both the upper and lower transits of Polaris, or any other close circumpolar star, and as the middle wire of the instrument, when exactly in the meridian, bisects the circle which the star apparently describes, round the polar point, in 24 sidereal hours, the time elapsed, during its traversing either the eastern or western semicircle, will be equal to 12 sidereal hours; but should the interval be greater or less, it is clear that the instrument deviates from the meridian. If the eastern interval is greater than the western, the plane in which the instrument moves from the zenith to the north of the horizon, is westward of the true meridian, and *vice versâ*, if the western interval is greatest. Having the difference of the interval from 12 hours, the quantity of deviation measured on the horizon may be computed by the following formula, the latitude of the place, and the polar distance of the star, being both supposed to be known, at least approximately.

$$\text{Deviation} = \log. \frac{\Delta}{2} + \log. \sec. L + \log. \tan. \pi - 20:$$

in which expression Δ = the difference of the intervals from 12^h (reduced to seconds)

π = the polar distance of the star

L = the latitude of the place.

This formula, in words, gives the following practical rule: Add together the log. of half the difference of the intervals from 12 hours in seconds, the log. secant of the latitude, and the log. tangent of the polar distance of the star: the sum, rejecting 20 from the index, will be the log. factor of deviation, which may be converted into arc by multiplying it by 15.

The correction of this error may be effected by turning the screw, *a*, if the angular value of one revolution be known, unless the instrument possesses an azimuth circle, by which the telescope may be set exactly that quantity from its present position. But if the quantity of motion to be given to the adjusting-screw, *a*, is not a matter of certainty, the observer, after ascertaining the difference of the intervals, must make the adjustment which he considers sufficient, and again proceed to verify it by observation, until, by continued approximation, he succeeds in fixing his instrument correctly in the meridian.

The above method of determining the instrumental deviation, is wholly independent of the tabulated place of the circumpolar star, but it assumes some knowledge of the rate of the time-keeper, and the *perfect stability* of the instrument for twelve hours; a condition which is rarely to be obtained, except in a regular observatory. The method is still further limited in practice, by the uncertainties of the weather, and the want of stars sufficiently bright to be observed in the daytime, (Polaris being the only star in the northern hemisphere fit for the purpose, and there is no similar star in the southern.) There are, however, two methods almost as good as the preceding, which depend on the *tabulated* places of the stars only. These will now be explained.

Take two *well-known* circumpolar stars, the nearer the pole the better, differing about twelve hours in right ascension, and observe one above and the other below the pole. Now it is evident, that any deviation of the instrument from the meridian will produce *contrary* effects upon the observed times of transit, exactly as in the upper and lower culmination of the same star. Hence, the time which elapses between the two observations will differ from the time which should elapse according to the catalogue, by the *sum* of the effects of the deviation upon the two stars. Compute what effect a deviation of 15″ will produce on the interval, then the difference between the observed interval and computed interval, divided by the quantity thus computed, will be the factor of deviation to be used for correcting transits observed the same night; or, if the deviation itself be required for altering the position of the instrument, multiply this factor by 15, the result will be the deviation to the east or west of the north in seconds of space.

The effect produced on the interval by a deviation of 15″, is to be computed as follows: let π be the polar distance of the upper star, π' that of the star sub-polo, λ the co-latitude of the place: then the effect in time of a deviation of 15″ is, for the upper star $\frac{\sin.(\lambda-\pi)}{\sin.\pi}$ and for the star sub-polo $\frac{\sin.(\lambda+\pi')}{\sin.\pi'}$, acting contrary ways upon the time of transit of each star respectively, and hence affecting the interval by their sum, or by $\frac{\sin. \lambda \sin. (\pi + \pi')}{\sin. \pi \sin. \pi'}$. Hence the factor for instrumental deviation $= \frac{\sin. \pi \sin. \pi'}{\sin. \lambda \sin. (\pi + \pi')} \times$ the difference between the observed and computed intervals. When $\pi = \pi'$, or the same star is observed at the upper and lower culmination, this factor becomes $\frac{\tan. \pi}{2 \sin. \lambda} \times$ the difference.

Practical Rule. To the log. of the difference in seconds between the tabulated and observed interval, add the log. sines of the polar distances of the two stars, the log. secant of the latitude, and the log. co-secant of the sum of the two polar distances, reject 40 from the index, and the result will be the log. factor of the deviation, (to be used according to the formula, page 86, in correcting the transits of all stars observed the same night.) And, as before observed, when it is intended to correct the position of the instrument, this quantity, multiplied by 15, will give the deviation from the meridian in space to the east or west of the north.

In determining the *direction* of the deviation, it must be recollected, that when the deviation is to the east, the star above pole passes too early, and that below pole too late, and therefore, if the upper star precedes, the interval is increased, but if the lower precedes, then *vice versâ*. When the deviation is to the west, the star above pole passes too late; while the star below pole passes too early. Hence, if the former precedes, the interval is diminished, and *vice versâ*.

EXAMPLE.

At page 79 we have inserted an extract from the Greenwich Observations for 1834, page 10: we shall take for an example the stars Cephei 51 *Hev.* and δ Urs. Min. S. P.

March 18th, 1834.

		Observation.		Naut. Alm.
		H. M. S.		H. M. S.
Cephei 51 *Hev.* . . .	=	6 20 59,00	.	6 20 19,61
δ Urs. Min. S. P. . .	=	6 26 32,50	.	6 25 51,24
Interval observed .	=	5 33,50	.	5 31,63
„ tabulated .	=	5 31,63		
Diff. of Intervals .	=	1,87	log.	0·27184

			(Brought forward)	
Diff. of Intervals .	=	1,87	log.	0·27184
		° ′ ″		
π the P. D. of Cephei .	=	2 43 50	sine .	8·67796
π′ „ δ Urs. Min. .	=	3 25 4	sine .	8·77536
Latitude . . .	=	51 28 39	secant	0·20564
(π + π′) sum of the Polar distances .	=	6 8 54	co-sec.	0·97020
			s 0,080 =	8·90100
Multiply by . . .			15	
Deviation from the meridian in space	=		1,200	

In determining the direction of the above deviation, we must observe according to our precepts, that the upper star precedes, and the observed interval is greater than the tabulated interval, therefore the deviation is to the east of north.

This method may now be practised very conveniently, as the apparent places of δ Ursæ Minoris and Cephei 51 *Hev.* are given in the Nautical Almanac. In like manner, Polaris may be combined, though less advantageously, with the stars of the Great Bear.

Again, Polaris, or any close circumpolar star, the place of which is accurately known, may be combined with any star distant from the pole. The simplest mode of considering this is, that the star which is distant from the pole, gives the *time* or error of the time-keeper; and again, if Polaris gives the same error, that the instrument must be in the meridian,* the formula for computation is the same as in the next following method, commonly called that of high and low stars, but is much more accurate.

The last method we shall speak of for correcting the position of the instrument, is by observing the transit of any two stars differing from each other considerably in declination, (at least 40°) and but little in right ascension. The nearer the right

* Persons desirous of avoiding computation, and who do not want the greatest possible accuracy, may proceed conveniently thus: Get the error of the time-keeper from stars as near the zenith as may be, levelling with the utmost care before each observation, and reversing the instrument once during the series. By taking a mean of the whole, an excellent error of the time-keeper will be found, unaffected by errors of deviation or collimation, and, if the levelling has been performed with all care, of inclination too. With this error, find what time, by the time-keeper, Polaris, δ Ursæ Minoris, or Cephei 51 *Hev.*, should transit, and adjust the azimuthal screws accordingly. If the observer has made out, as he always ought to do, the time between each wire, and the middle wire, as well as the value of the revolutions of his adjusting-screw, he may compute the time for *each* wire, and examine his success at each, as the star passes through the field of the telescope. It is necessary to add, that the level should always be examined after touching the azimuth-screws.

ascensions of the stars are to each other the better, as this prevents the possibility of any error arising from a change in the rate of the time-keeper affecting the observations. And as the apparent places of one hundred principal stars are now given in the Nautical Almanac for every tenth day, it will be better to select a pair from thence, which will save the trouble of computing their apparent right ascensions; and, as many suitable pairs are contained therein, it will seldom happen, but that the passage of some of them will occur at a convenient time for observation.

The times of the transits of the two stars being observed (without regard to the *error* of the time-keeper), the deviation of the instrument from the plane of the meridian may be thus determined: Take the difference between the observed passages of the two stars, and also the difference of their computed right ascensions (calling the differences + when the lower star precedes the higher, and *vice versâ*); and if these differences be exactly equal, the instrument will be correctly in the plane of the meridian; if they are not equal, their difference, that is to say, the difference of the observed times of transit, *minus* the difference of the computed right ascensions, will point out a deviation from that plane, to the eastward of the south when it is +; and west when it is —. As an example, let us take the following:

	Observed Time.		Apparent A.R.
	H. M. S.		H. M. S.
Higher star	5 46 51,91	...	5 46 53,50
Lower ,, ...	6 37 25,66	...	6 37 33,66
Difference............ =	— 50 33,75	...	— 50 40,16
Subtract diff. of A.R. =	— 50 40,16		
	+ 6,41		

+ 6,41 = the difference of time *minus* the difference of right ascension, which being + shows that the instrument deviates to the eastward of the south point of the horizon. It is evident that a high star will be less affected by deviation, than one in any other situation, and that a star between the pole and zenith will be *differently* affected from a star south of the zenith, it being observed sooner than it ought when the latter is observed later, and *vice versâ*.

The deviation in azimuth may now be computed from the following formula:

$$\text{Deviation in azimuth} = \text{D. sin. } \pi \text{ sin. } \pi' \text{ co-sec. } (\pi \mp \pi') \text{ sec. L.}$$

In which D represents the difference of times *minus* the difference of right ascensions; π and π' the polar distances of the higher and lower stars, and L, as before, the latitude of the place of observation.

This formula, in words, gives the following rule: To the log. of the difference of times *minus* the difference of right ascensions, add the log. sin. of the polar distance of the higher star, the log. sin. of the polar distance of the lower star, the log. cosecant of the difference or sum of the polar distances of both the stars, (the difference when they are both above the pole, and the sum when one is above and the other below the pole,) and the log. secant of the latitude: the sum will be the log. of the azimuthal deviation, which multiplied by 15 will be the deviation in arc.

As a complete example, let us take a high and low star, from the same day's work at Greenwich that we took our former example from, and see how nearly alike the deviation comes out by the two calculations.

March 18th, 1834. (See page 79.)

	Observed Time.		Apparent A.R. from Naut. Alm.
	H. M. S.		H. M. S.
Higher star, Cephei 51 *Hev.*	6 20 59,00	...	6 20 19,61
Lower „ Sirius	6 38 30,88	...	6 37 49,76
Difference............ =	— 17 31,88	..	— 17 30,15
Subtract diff. of A.R. .. =	— 17 30,15		
	— 1,73		

— 1,73 = the difference of time *minus* the difference of right ascension, which being — shows that the deviation is to the west of south, agreeing with our former determination, (page 75) viz., east of north. Let us now compute the deviation in azimuth by our last formula, and see how nearly they agree.

Difference of intervals	$1^s,73$	..	log. ..	0·23805
π the P.D. of Cephei	$2° \; 43' \; 50''$	..	sine ..	8·67796
π' „ Sirius	106 29 50	..	sine ..	9·98174
$(\pi'—\pi)$ the diff. of Polar distances		..	co-sec.	0·01266
L = the latitude = 51° 28′ 39″		..	sec. ..	0·20564
			$0^s,131$ =	9·11605
Multiply by		......	15	
The azimuthal deviation in arc, west of south		=	1,965	

"The time employed in making these observations is supposed to be sidereal time, therefore, if a clock or watch be used which marks mean solar time, the interval between the observations must be corrected accordingly." This correction is made by adding to the difference of the observed times, the acceleration of the fixed stars for that interval, (Table IV.,) which will convert that portion of *mean* into an equivalent portion of *sidereal time;* so that by means of this correction it will be indifferent whether the clock shows sidereal or mean time.

"If, before or after the passage of the stars, the telescope be pointed to the horizon and compared with some object there, a meridian mark may be set up, which may be corrected from time to time by subsequent observations on various stars similarly situated, and when once *correctly* fixed, it will serve to verify both the meridional position of the instrument, and the adjustment of the collimation."

Having, by means of the previous adjustments, made the line of collimation describe a great circle passing through the zenith of the place, and the north and south points of the horizon, the instrument will be in a fit state for making observations. We have said that the telescope contains five vertical and two horizontal wires, placed a short distance from each other; these last are intended to guide the observer in bringing the object to pass across the middle of the field, by moving the telescope until it appears between them: the centre vertical is the meridional wire, and the instant of a star's passing it will be the time of such star's being on the meridian; but as, in noting the time, it will not often happen that an exact second will be shown by the clock, when the star is bisected by the wire, but it will pass the wire in the interval between two successive seconds, the observer must, therefore, whilst watching the star, listen to the beats of his clock, and count the seconds as they elapse; he will then be able to notice the space passed over by the star in every second, and consequently its distance from the wire at the second before it arrives at, and the next second after it has passed it, and with a little practice he will be able to estimate the fraction of a second at which the star was on the wire, to be added to the previous second: thus, suppose the observer counted 4, 5, 6, 7, 8 seconds, whilst watching the passage of a star, which passed the wire between the 7th and 8th, at which times it appeared equally distant on each side of it, the time of the transit would then be $7^s,5$; but if it appeared more distant on one side than the other, it would be $7^s,3$, or $7^s,7$, &c., according to its apparent relative distance from the wire.

This kind of observation must be made at each of the five wires, and a mean of the whole taken, which will represent the time of the star's passage over the mean or meridional wire. The utility of having five wires instead of the central one only will be readily understood, from the consideration that a mean result of several observations is deserving of more confidence than a single one; since the chances are, that an error which may have been made at one wire will be compensated by an opposite error at another; thus destroying each other's effect, the mean result will come out very nearly the same as the observation at the middle wire, if they are made with any tolerable degree of accuracy, and if the intervals of the wires are uniform.

Example of the Greenwich mode of registering Transit Observations.

Date.	Illuminated End—East. I.	II.	Merid. Wire	IV.	V.	Mean of Wires.	Clock. Error.	Clock. Rate.	No. of Days.	Object.
	S.	S.	M. S.	S.	S.	H. M. S.	M. S.	S.		
1834 Mar. 18	36,1	54,4	50 12,7	31,1	49,5	23 50 12,76	..	..	..	⊙ 1 Limb.
	45,2	3,5	52 21,9	40,3	58,6	23 52 21,90	..	..	..	⊙ 2 Limb.
	..	..	..	..	..	23 51 17,33	+0 41,32	+0,01	4	⊙ Centre.
	14,1	40,5	5 6,6	32,7	59,1	5 5 6,60	+0 41,31	+0,05	3	Capella.
	..	..	20 59,0	..	..	6 20 59,00	..	..	..	Cephei 51 *Hev.*
	..	..	26 32,5	..	..	6 26 32,50	..	..	..	δ Urs. Min. S.P.
	52,6	11,8	38 30,7	50,1	9,2	6 38 30,88	+0 41,13	0,00	3	Sirius.

The annexed Table is an example of the Greenwich mode of registering observations made with a transit-instrument.

The heading at the top of the columns sufficiently explains the nature of their contents. The error of the clock from sidereal time is obtained, by taking the difference between the mean of the wires, and the apparent right ascension of the object as given in the Nautical Almanac; and the daily rate is the difference of such errors, divided by the number of days elapsed between the observations. In observing the sun, the times of passing of both the first and second limb over the wires are observed and set down as distinct observations, the mean of which gives the time of the passage of the centre across the meridian, as is shown in the annexed example. The wires of the instrument are generally placed by the maker at such a distance from each other, that the first limb of the sun shall have passed all of them before the second limb arrives at the first, and the observer can thus take the observations without hurry or confusion.

One limb only of the moon can be observed, except when her transit happens to be within an hour or two of her opposition; and in observing the larger planets, the first and second limb may be observed alternately over the five wires; that is to say, the first limb over three wires, viz., the first, third, and last; and the second limb over the second and fourth; which being reduced in the same manner as the observation of the sun, will give the meridional passage of the centre. When an observation at one or more of the wires has been lost, it is impossible to take the mean in the same way as in a perfect observation. If the centre wire is the one that is deficient, the mean of the other four may be taken as the time of the meridional passage, or the mean of any two equally distant on each side of the centre, (supposing the interval of the wires to be equal;) but when any of the side wires are lost, and indeed under any circumstance of deficiency in the observation, the most correct method of proceeding is as follows: By a considerable number of careful observations over all the wires, the equatorial interval between each side wire and the centre one is to be deduced and set down for future use. Then, when part of the wires only are observed, each wire is to be reduced to the mean, by adding or subtracting, as the case may be, to the time of observation, the equatorial interval between that wire and the centre wire, multiplied by the secant of the declination of the star, as in the following rule.

To the log. of the equatorial interval (from the wire at which the observation was made to the centre) add the log. secant of the star's declination (or co-sec. of its polar distance,) the sum, rejecting ten from the index, will be the log. of the interval from the wire at which the transit was taken, to the centre wire, which being added to observations made at the first or second wire, or subtracted from those made at the fourth or fifth, will give the time of the star's passing the meridional wire.

The equatorial intervals of the wires may readily be computed by the following rule, from observations made upon any star whose declination is known. To the log. of the interval occupied by the star in passing from any wire to the centre wire, add the log. cosine of the star's declination (or sine of its polar distance;) the sum, rejecting ten from the index, will be the log. of the equatorial interval, which being determined for each wire, from observations of a number of stars having different declinations, the mean will be a very correct result. The equatorial intervals of the wires of the transit at the Royal Observatory, were found to be,

				s.
From the first wire	to the third		=	36,647
„	second	„	=	18,305
„	fourth	„	=	18,309
„	fifth	„	=	36,606

The middle wire at Greenwich coincides with the mean of the wires, the intervals being very nearly equal, but when this is not the case, the observer must correct the mean of the wires for the difference from the centre wire, to obtain a correct mean; the correction to be applied to the mean of the wires may be computed as follows: divide the difference between the sum of the first two and sum of the last two equatorial intervals by 5, and to the log. of the quotient add the log. co-secant of the polar distance of the star; the sum will be the log. of the correction required, *plus* if the sum of the two first intervals is greater than the second, otherwise *minus*. Such inequality in the intervals should never be allowed to remain, unless circumstances prevented their rectification.

In regular observatories, the transit-instrument is employed, not only for the determination of time, but in forming catalogues of the right ascensions of the fixed stars, and other important operations in astronomy; purposes for which instruments of a superior class, and fixed in their respective places, are required. But, from the small size and low optical power of the portable transit-instrument, it can be applied with good effect only to the determination of time, and of the longitude by observations of the moon and moon-culminating stars. The Nautical Almanac contains the true apparent right ascension of the sun, and of one hundred of the principal fixed stars; that is, the sidereal time when each of them, respectively, is on the meridian, or on the centre wire of a properly adjusted transit-instrument; and if the instant when a star so passes the central wire, be noted by a clock correctly adjusted to sidereal time, the time shown by the clock will be the right ascension of the star as given in the Almanac. The difference therefore between the time shown by a clock, and such right ascension, will be the error of the clock from sidereal time + (or too fast) when the clock time is greater than the right ascension, and — (or too slow) when it is less. Thus, on March 18th, 1834, (page 79,)

		H.	M.	S.
The observed passage of Capella by clock . .		5	5	6,60
Right ascension by Naut. Alm.		5	4	25,29
Clock error	+		0	41,31

In the same manner the error of the clock is deduced from an observed transit of the sun's centre, the time of which, as before shown, is derived from a mean of the observations of the first and second limbs; but when, from intervening clouds or other circumstances, one limb only can be observed, the passage of the centre may be found, by adding or subtracting *the sidereal time of the sun's semidiameter passing the meridian*, as given in the Nautical Almanac, according as the first or second limb may be observed.

If the clock error be determined in this manner from a number of observations each day, the mean of the whole will probably be a very accurate determination of the error for the mean of the times at which the observations were made. In like manner the mean daily rate may be found by taking the difference between the errors as determined by the same object from day to day; and if more than one day has elapsed between the observations, dividing the change in the error by the number of days elapsed; the rate, when the clock is too fast, will be + (or gaining) when the second error is greater than the first, and — (or losing) when the second error is the least; and *vice versâ*, when the clock is too slow.

When a clock or chronometer, showing mean solar time, is employed, its error from such time may be found, by computing the mean time of the passage of the object over the meridian of the place, and the difference between such mean time, and the observed time of the object's meridian passage, will, as before, be the error of the clock from mean time.

The following is the method of computing the mean solar time of the transit of a star across the meridian.

From the right ascension of the star, subtract the sidereal time at mean noon for the given day, taken from the Nautical Almanac, (adding 24 hours to the former when the latter exceeds it) the remainder is the sidereal interval after noon of that day. From this, subtract the acceleration of sidereal upon mean time, and the result is the required mean solar time of the passage. As an example, suppose it were required to find the mean time of the passage of Capella on March 18th, 1834:

		H.	M.	S.
Right ascension of Capella (+ 24 hours)		29	4	25,29
Sidereal time at mean noon		23	42	15,64*
Sidereal interval, past noon	=	5	22	9,65
Acceleration of sidereal on mean time for the interval	=		—	52,78
Mean time of passage	=	5	21	16,87

The acceleration of sidereal on mean time is to be taken from Table III.; thus, in the above example:

		M.	S.
Acceleration for 5 hours		0	49,148
„ 22 minutes			3,604
„ 9 seconds			0,025
„ 65 hundredths . . .			0,003
For the whole interval	=	0	52,780

* The sidereal time as given in the Nautical Almanac, is for mean noon at Greenwich, and therefore must be corrected for any other meridian, as directed in the explanation of the articles, given at the end of the Almanac.

Table III. will not answer for performing the reverse operation, viz., converting a portion of mean solar time into a corresponding portion of sidereal time ; Table IV. must be employed for this purpose, adding to the given portion of mean time the quantity taken from the table corresponding thereto, and the sum will be an equivalent portion of sidereal time. As an example we will take the above case of Capella.

		H.	M.	S.
Mean time		5	21	16,87
Table IV.	+			52,78
Sidereal interval		5	22	9,65
Sidereal time at mean noon		23	42	15,64
		29	4	25,29
	—	24		
Sidereal time of the star's passage, or its right ascension		5	4	25,29

The method of taking out the correction from Table IV. is exactly similar to that given in the above example for Table III.

To find the error of a clock or chronometer intended to show mean time from an observed transit of the sun, nothing more is necessary than to apply the equation of time to 24 hours, and the difference between the result and the time of the sun's transit, as shown by the chronometer, is the error of the chronometer for mean time ; + when the chronometer time is the greater, and — when it is the less.

From the description which has been given of the method of bringing a transit-instrument into a state of perfect adjustment, it might be inferred that it is essential it should be strictly so, to obtain accurate results from the use of it. It is certainly desirable that the adjustments should be examined and rectified as often as possible, as doing so ultimately saves the labour of computing the corrections to be applied to each observation, on account of the errors in the position of the instrument. But in some established observatories, where large instruments are employed, it is not attempted to put them in perfect adjustment, but the amount of the various derangements is ascertained from time to time, and the observations corrected accordingly. The adoption of this method, with so small an instrument as the one which we have been describing, where the adjustments are easily examined and corrected, will give indeed more accurate results, but, on account of the greater trouble, is not perhaps to be generally recommended ; we shall, nevertheless, introduce in this place, an account of the method of computing these corrections, that persons possessing transit-instruments may adopt which method they think proper.

The first correction is for the deviation of the line of collimation: the amount of the error may be determined by a micrometer attached to the eye-end of the telescope, by which, when the telescope is directed towards any distant object, the angular distance of that object from the central wire is measured in revolutions and parts of the micrometer-screw. The instrument is then reversed, and the distance of the same object from the central wire again measured, when half the difference of the measures is the error in collimation; and the angular value of a revolution of the screw being known, the corresponding value of the error is likewise known. The correction on account of this error to be applied to the time of each observation may be computed from the following formula:—

$$\text{Correction} = \frac{c}{15} \text{ co-sec. } \pi$$

c = the error of collimation + if the deviation is toward the east.

π = (as before) the polar distance of the star.

Hence we have in words this rule: To the log. of the deviation in collimation, add the log. co-secant of the polar distance of the star, and the arithmetical complement of the log. of 15: the sum will be the log. of the correction in time required.

The next correction to be considered, is that arising from a want of horizontality in the axis. The spirit-level, which we described as striding across the instrument and resting on the pivots, determines the amount of the inclination of the axis, and also, as we have seen, enables the observer to correct it. Above the glass tube, and parallel to its length, is placed a fine graduated scale, the reading of which points out the number of seconds in arc that the pivots deviate from the true level, shown by the air-bubble receding from the centre towards that pivot which is the highest; but as it is necessary, when correcting for the adjustment, to remove half the error, by giving motion to the little screw on the level itself, so, for the same reason, in finding the measurement of the error, it is necessary to reverse the level on the axis, and read the scale at each extremity of the air-bubble in both its positions; that is, with the same end of the level on both the east and west pivots alternately,—and the difference of the sums of the two readings divided by the number of readings will be the amount of deviation. This may be illustrated by the following example, in which the divisions on the scale represent seconds.*

* The value of the divisions of the scale may be had from the maker.

Readings of the Scale.

East End.		West End.
$''$		$''$
109,0	. . . .	69,6
109,0	. . . .	69,8
108,8	. . . .	69,9
	Level Reversed.	
69,0	. . . .	109,0
68,6	. . . .	108,9
69,1	. . . .	109,0
533,5	Sums	536,2
		533,5

Divide by the number of observations, 12) 2,7

$\frac{1}{2}$ difference $=$ 0,23 $=$ the amount of deviation in arc, showing that the west end of the axis is higher by that quantity than the east end, since the sum of the western readings is greater than the sum of the eastern. This quantity, divided by 15, will give the proper factor for inclination. It is more convenient that the scale should be divided into units, each of which is $15''$.

Having in this manner determined the inclination of the axis by the level, the correction to be applied to the time of observation of any star made during the existence of that error, may be computed from the following formula :—

$$\text{Correction} = b \cos. (\pi - \lambda) \text{ co-sec. } \pi$$

b = the factor for the inclination of the axis + if the west end be too high.

π = the polar distance of the star.

λ = the co-latitude of the place.

This formula in words gives the following practical rule. To the log. of the factor for inclination of the axis, add the log. co-secant of the polar distance, and the log. co-sine of the difference between the polar distance and the co-latitude : the sum —20 will be the log. of the correction in time required.

We have already explained the manner of ascertaining the azimuthal deviation of the instrument from the plane of the meridian, page 72, &c. The correction to be added algebraically to the observed time of transit of any star whilst the instrument so deviates, may be computed from the following formula :—

$$\text{Correction} = a \sin. (\pi - \lambda) \text{ co-sec. } \pi$$

in which a = the factor for azimuthal deviation, + when the instrument deviates to the eastward of the south meridian.

π = the polar distance of the star.
λ = the co-latitude of the place.

This formula in words gives the following:—

Rule.—To the log. of the factor for azimuthal deviation, add the co-secant of the polar distance, and the sine of the difference between the polar distance and co-latitude: the sum will be the log. of the correction required.

As an example, let us take the star ϵ Bootis. (Pearson's Astron. vol. ii. p. 344.)

		H. M. S.
Observed time of transit . . .	=	14 35 4, 86
Error of collimation . . 12″ or	= +	0, 80
Inclination of the axis	= —	1, 75
Deviation of Instrument in azimuth .	= —	4,737

The errors are in units, each of which = 15″

		° ′
Polar distance . . .	=	62 12
Co-latitude . . .	=	38 27

The correction for the Collimation.

Deviation = + 0,80 log. . . .	= + 9·90309
Polar dist. 62° 12′ co-secant . . .	= + 0·05326
Correction = + 0^s,904 . log.	= + 9·95635

The correction for the Level.

Deviation = — 1,75 log. . . .	= — 0·24304
Polar dist. 62° 12′ co-secant . . .	= + 0·05326
Polar dist. minus co-lat. 23° 45′ cos. .	= + 9·96157
Correction = — 1^s,811 . log.	= — 0·25787

The Correction in Azimuth.

Deviation = — 4,737 log...	= — 0·67550
Polar dist. 62° 12′ co-secant	= + 0·05326
Polar dist. minus co-lat. = 23° 45′ sine	= + 9·60503
Correction = — 2^s, 157 .. log.	= — 0·33379

Now apply the sum of these corrections to the observed time of the star's transit, and the actual time of transit will be obtained as correctly as if the instrument had been in a state of perfect adjustment when the observation was made.

		H.	M.	S.
Observed time of transit..	=	14	35	4,860
Correction for the collimation ..	=	+		0,904
,, ,, level	=	—		1,811
,, in azimuth	=	—		2,157
Corrected observation......	=	14	35	1,796
Computed right ascension ...	=	14	37	28,910
Clock slow on sidereal time .	=		2	27,114

Besides the determination of time, the portable transit-instrument may be successfully employed in determining the longitude. The Nautical Almanac contains, for each lunation, a list of the right ascensions and declinations of the moon-culminating stars, whose meridional transits being observed, together with that of the moon, at any two places, the differences of right ascension thus obtained between the moon's illuminated limb and each of those stars, form the data required for computation. "If the moon had no motion, the difference of her right ascension from that of a star would be the same at all meridians, but in the interval of her transit over two different meridians, her right ascension varies, and the difference between the two compared differences exhibits the amount of this variation, which, added to the difference of meridians, shows the angle through which the westerly meridian must revolve before it comes up with the moon; hence, knowing the rate of her increase in right ascension, the difference of longitude is easily obtained."

The necessity of having recourse to actual observation of the same stars at the two places, in order to obtain the longitude, may soon be dispensed with, since their apparent right ascensions are given in the Nautical Almanac. At present, however, and until the places of the moon-culminating stars are perfectly well known, corresponding observations are required for the accurate determination of differences of longitude.

The difference of longitude between the stations, is supposed to be approximately known, or may be got near enough for an approximation, by dividing the difference between the observed and computed right ascension of the moon's bright limb by the hourly motion given in the Nautical Almanac.

The formula for computation, with the necessary explanation, may be found in the Memoirs of the Royal Astronomical Society, vol. ii. p. 1, &c. Availing myself of the kind permission of Mr. Riddle, I am enabled to insert his method of performing the computation, together with Table XXXIII. of his valuable treatise on Navigation.

PRACTICAL RULE.

To the estimated longitude in time, add the correction from Table XI., and apply the sum to the time of the moon's passing the meridian of Greenwich, as given in the Nautical Almanac, adding if the longitude is *west*, or subtracting it if east; and the sum or the remainder will be the approximate Greenwich date for the moon's passing the given meridian.

Find the moon's right ascension, both for this time and the time of her passing the meridian of Greenwich, and divide the difference of her right ascensions by the hours, &c., in the difference of these times, and the quotient will be the mean hourly change of the moon's right ascension in the interval, which is the argument of Table V.

Take also the declinations roughly for the same two times.

With the mean of these declinations, and the change of the moon's semidiameter, take the correction from Table VI. and apply it to the interval between the transits of the star and the moon's bright limb, as observed at or computed for the more westerly meridian.

Again, with the mean of the declinations take the corrections from Table XII., and multiplying it by the degrees in the moon's change of declination, apply the product as a second correction to the western interval.

The following formula will show the signs with which these corrections are to be applied.

Sign of First Correction.

	Moon.	Limb Obsd.	Correct.
Moon's semidiam. increasing	preceding star	W	—
		E	+
	following star	W	+
		E	—
Moon's semidiam. decreasing	preceding star	W	+
		E	—
	following star	W	—
		E	+

Sign of Second Correction.

	Moon.	Limb Obsd.	Correct.
Moon's declination increasing	preceding star	W	+
		E	—
	following star	W	—
		E	+
Moon's declination decreasing	preceding star	W	—
		E	+
	following star	W	+
		E	—

The change of semidiameter here spoken of is that taken from the Ephemeris, without augmentation for altitude.

The interval at the more westerly meridian being thus corrected, call the seconds of the differences of the intervals, A;

or, if more than one star has been observed, call the seconds in the mean of the differences of the corresponding intervals A.

If either of the intervals be in mean time, add to it its 360th part diminished by the 70th part of itself, and the sum will be the corresponding interval in sidereal time. And if both are in mean time, reduce their difference to sidereal time by the same rule. Table IV. may also be used for this purpose.

If the moon precede the star at the easterly, and follow it at the westerly meridian, the *sum* of the intervals instead of the *difference* will be A.

Then *add* the *logarithm of the seconds in* A, the difference of the *sidereal intervals* to the *logarithm* from *Table V.*, and the sum will be the *logarithm* of the difference of longitude in *seconds of time.*

Note. The parts for hundredths in Table V. are found in the column of ‘ parts’ opposite the corresponding tenths. Thus, for $1^m\,42^s,57$, the log. for $1^m 42^s,5$, is 1·534256, and the part for seven hundredths is 304—whence the log. is 1·533952. Striking off the figures on the right, in the column of ‘parts,’ the remaining figures on the left are parts for thousandths.

EXAMPLE.

December 8th, 1834.

Star, &c.	Clock Transit, observed at Greenwich.	Rate of Clock.	Clock Transit, observed at Cambridge.	Rate of Clock.
	H. M. S.	S.	H. M. S.	S.
96 Aquarii ..	23 10 58,18	— 0,68	23 10 14,40	— 2,56
n Piscium ..	23 39 35,32		23 38 51,72	
☽ ’s 1st Limb	23 47 29,86		23 46 45,52	
s Piscium ..	23 57 1,18		23 56 17,53	
n Ceti	0 21 45,08		0 21 1,32	

First find the mean intervals between the passage of the stars and the moon at both places, thus :—

Greenwich Intervals.		Cambridge Intervals.	
M.	S.	M.	S.
36	31,68	36	31,12
6	54,54	7	53,80
9	31,32	9	32,01
34	15,22	34	15,80

Intervals corrected for Rate.

36	31,70	36	31,18
7	54,54	7	53,81
9	31,32	9	32,03
34	15,23	34	15,85

Diff. Intervals.
·52
·73
·71
·62
Mean . . . ·65

On December 8th, 1834, the moon passed the meridian of Greenwich at $6^h\ 40^m$, the declination being then about 7°, and it would be about one thousandth of a degree different at Cambridge; and the 1000dth part of ·134 (the number corresponding to 7° of declination in Table XII.) is too small a quantity to be worth attention. This also is the case with the effect of the change in the moon's semidiameter, the change being not more than a thousandth of a second of space; and the effect of that small change on the time of the moon's transit being clearly beyond the reach of notice in ordinary observers. The Nautical Almanac gives the following:—

					M.	S.
Hourly change of ☽'s R.A, from	5 hours to	6	.		1	50, 19
„ „	6 „	7	.		1	50, 03
„ „	7 „	8	.		1	49, 87

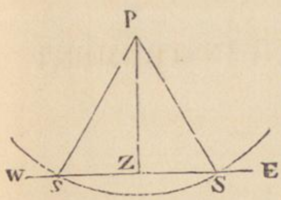
Hence, at $6^h\ 40^m$ the hourly rate of change would be about $1^m\ 50^s,08$.

M. S.		
1 50,08	Table V.	1·502334
,65	log.	= 9·812913
Longitude of Cambridge in time $21^s,3$		1·315247

The longitude of the Cambridge Observatory has been determined by Professor Airy to be $23^s,5$. The reader may perhaps be surprised, that the above result differs 2^s, 8 from it; but it may be remarked, that by this method of finding longitude, it is absolutely necessary, that a great number of results be taken as a satisfactory determination. This arises mostly from the errors made in observing the transit of the moon's limb, which, it is well known to practical men, is a very difficult observation to make correctly; and a very small error in the observation makes a considerable one in the final result: supposing the transits of the stars to have been observed *perfectly correct*, yet, if an error of only two tenths of a second be made in that of the moon's limb at either Observatory, the longitude deduced from such observations, would be incorrect to the amount of 6 seconds in time, at a mean rate of the moon's motion. When both limbs of the moon can be observed at both Observatories, which can only be the case when she is near the full at the time of transit, a better result can be obtained.

There is a mode of finding the latitude by the transit-instrument, pointed out by Professor BESSEL, and used with great success in the Russian survey, which we will now explain in some detail, as the method is not so commonly known or practised in this country as it deserves to be.

Place the transit-instrument with its supports north and south, so that the telescope when pointed to the horizon looks due east and west. Observe the passage of a well-known star over the middle wire when the telescope is pointing east, and again, observe the passage of the same star over the middle wire when the telescope is pointing west, noting the time carefully. The star should be near the zenith, (such a star as γ Draconis, for instance, in the latitude of London, and for a degree or two to the northwards,) as the observations take less time, and are therefore more independent of the timekeeper employed; the method is also more accurate when the star is near the zenith than when otherwise.



In the accompanying figure, P is the pole, Z the zenith, E Z W the prime vertical passing through the east and west points, the dotted line S s the path of the star; all seen as projected on the horizon from a point above Z. Then in the right-angled spherical triangle, P Z S, P S is the north polar distance of the star, P Z the co-latitude, and the angle Z P S, half the time elapsed from S to *s*, therefore, tan. P Z = tan. P S × cos. Z P S.

Let Δ'' = half the interval in time reduced to arc between the two transits of the star over the prime vertical, (a circle which passes through the zenith, and east and west points of the horizon.)

π = the N. P. D. of the star (taken from the Nautical Almanac.)

λ = the co-latitude of the place.

then $\tan. \lambda = \tan. \pi \cos. \Delta''$

or in words — to the log. tangent of the star's N.P.D. add the log. co-sine of half the time elapsed, and the sum — 10 will be the log. tangent of the co-latitude required.

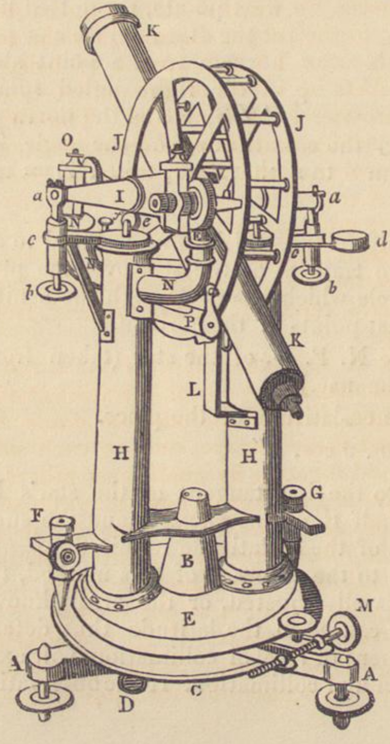
It is essential to the accuracy of this method, that the instrument should be well adjusted, or the errors known and allowed for. The error caused in the latitude thus determined, by the want of adjustment of level or collimation, will exactly equal the error of the level and collimation. If the observation be repeated on various nights, the telescope should be reversed. With these precautions, and a level of the best kind, the latitude may be obtained within a second or two, if the place of the star is sufficiently well known; and *differences* of latitude, whether the star be known or not.

To find the time when the star will come to the proper position for observation, viz., the prime vertical; first ascertain when the star will be on the meridian, by the method explained at page 82, then, by the following formula compute the time that would elapse during the passage of the star from the prime vertical to the meridian, or (referring to the above diagram) the angle, S P Z, which *time*, subtracted from that of the meridian passage, will give the time of the star's being on the prime vertical, or in the required position for making the first observation.

Formula, cos. S P Z (or Δ'') = tan. P.Z, cot. P S.

Practical Rule. To the log. tangent of the assumed co-latitude add the log. co-tangent of the star's polar distance, the sum will be the log. co-sine of half the elapsed time in arc, which divided by 15 will give the time required.

THE ALTITUDE AND AZIMUTH INSTRUMENT.



To the centre of the tripod, A A, is fixed the vertical axis of the instrument, of a length equal to about the radius of the circle; it is concealed from view by the exterior cone, B. On the lower part of the axis, and in close contact with the tripod, is centred the azimuth circle, C, which admits of a horizontal circular motion of about three degrees, for the purpose of bringing its zero exactly in the meridian; this is effected by a slow-moving screw, the milled head of which is shown at D. This motion should, however, be omitted in instruments destined for exact work, as the bringing the zero into the meridian is not requisite, either in astronomy or surveying; it is in fact purchasing a convenience too dearly, by introducing a source of error not always trivial. Above the azimuth circle, and concentric with it, is placed a strong circular plate, E, which carries the whole of the upper works, and also a pointer, to show the degree and nearest five minutes to be read off on the azimuth circle; the remaining minutes and seconds being obtained by means of the two reading microscopes, F and G: this plate, by means of the conical part B, (which is carefully fitted to the axis) rests on the axis, and moves concentrically with it. The conical pillars H H, support the horizontal or transit axis, I, which being longer than the distance between the centres of the pillars, the projecting pieces, *c c*, fixed to their top, are required to carry out the Y's, *a a*, to the proper distance, for the reception of the pivots of the axis; the Y's are capable of being raised or lowered in their sockets by means of the milled-headed screws, *b b*, for a purpose hereafter to be explained. The weight of the axis, with the load it carries, is prevented from pressing too heavily on its bearings, by two friction rollers on which it rests, one of which is shown at *e*. A spiral spring, fixed in the body of each pillar, presses the rollers upwards, with a force nearly a counterpoise to the superincumbent weight; the rollers on receiving the axis yield to the pressure, and allow the pivots to find their proper bearings in the Y's, relieving them, however, from a great portion of the weight.

The telescope, K, is connected with the horizontal axis, in a manner similar to that of the portable transit-instrument. Upon the axis as a centre, is fixed the double circle J J, each circle being close against the telescope, and on each side of it. The circles are fastened together by small brass pillars; by this circle the vertical angles are measured, and the graduations are cut on a narrow ring of silver, inlaid on one of the sides, which is usually termed the *face* of the instrument: a distinction essential in making observations. The clamp for fixing, and the tangent-screw for giving a slow motion to the vertical circle, are placed beneath it, between the pillars, H H, and attached to them, as shown at L. A similar contrivance for the azimuth circle is represented at M. The reading microscopes for the vertical circle, are carried by two arms bent upwards near their extremities, and attached towards the top of one of the pillars. The projecting arms are shown at N, and the microscopes above at O.

A diaphragm, or pierced plate, is fixed in the principal focus of the telescope, on which are stretched five vertical and five horizontal wires: the intersection of the two centre ones, denoting the optical axis of the telescope, is the point with which a terrestrial object is bisected, when observing angles for geodetical purposes. The vertical wires are used for the same purpose as those in the transit telescope, and the horizontal ones for taking altitudes of celestial objects. A micrometer having a movable wire is sometimes attached to the eye-end of the telescope, but it is not generally applied to instruments of portable dimensions. The illumination of the wires at night is by a lamp, supported near the top of one of the pillars, as at *d*, and placed opposite the end of one of the pivots of the axis, which being perforated, admits the rays of light to the centre of the telescope-tube, where falling on a diagonal reflector, they are reflected to the eye, and illuminate the field of view: the whole of this contrivance is precisely similar to that described as belonging to the transit-instrument.

The vertical circle is usually divided into four quadrants, each numbered, 1°, 2°, 3°, &c., up to 90°, and following one another in the same order of succession; consequently, in one position of the instrument, altitudes are read off, and with the face of the instrument reversed, zenith distances; and an observation is not to be considered complete, till the object has been observed in both positions. The sum of the two readings will always be 90°, if there be no error in the adjustments, in the circle itself, or in the observations.

It is necessary that the microscopes, O O, and the centre of the circle, should occupy the line of its horizontal diameter; to effect which, the up-and-down motion (before spoken of) by means of the screws, *b b*, is given to the Y's to raise or lower them, until this adjustment is accomplished. A spirit-level, P, is suspended from the arms which carry the microscopes: this shows when the vertical axis is set perpendicular to the horizon. A scale, usually showing seconds, is placed along the glass-tube of the level, which exhibits the amount, *if any*, of the inclination of the vertical axis. This should be noticed repeatedly whilst making a series of observations, to ascertain if any change has taken place in the position of the instrument after its adjustments have been completed. One of the points of suspension of the level is movable, up or down, by means of the screw, *f*, for the purpose of adjusting the bubble. A striding-level, similar to the one employed for the transit-instrument, and used for a like purpose, rests upon the pivots of the axis. It must be carefully passed between the radial bars of the vertical circle to set it up in its place, and must be removed as soon as the operation of levelling the horizontal axis is performed. The whole instrument stands upon three foot-screws, placed at the extremities of the three branches which form the tripod,* and brass cups are placed under the spherical ends of the foot-screws. A stone pedestal, set perfectly steady, is the best support for this as well as the portable transit-instrument.

Of the Adjustments.

The first adjustment to be attended to, after setting the instrument up in the place where the observations are to be made, is to set the azimuthal or vertical axis truly perpendicular to the horizon: the method of doing this is to turn the instrument about, until the spirit-level, P, is lengthwise in the direction of two of the foot-screws, when by their motion the spirit-bubble must be brought to occupy the middle of the glass-tube, which will be shown by the divisions on the scale attached to the level. Having done this, turn the instrument half round in azimuth, and if the axis is truly perpendicular, the bubble will again settle in the middle of the tube; but if not, the amount of deviation will show double the quantity by which the axis deviates from the vertical in the direction of the level; this error must be corrected, one half by means of the two foot-screws (in question,) and the other half by raising or lowering the spirit-level itself, which is done by the screw represented at *f*. The above process of reversion and levelling should be repeated, to ascertain if the adjustment has been correctly performed; for, as we before observed, when speaking of the transit-instrument, adjustments of every kind can be made perfect only by successive trials and approximations.

Next turn the instrument round in azimuth a quarter of a circle, so that the level, P, shall be at right angles to its former position; it will then be over the third foot-screw, which may be turned until the air-bubble is again central, if not already so, and this adjustment will be completed: if delicately performed,

* The foot-screws are sometimes made in the following ingenious manner, as described by Mr. Troughton, in the Memoirs of the Astronomical Society, vol. i. p. 37. "Each of the three screws is double, that is, a screw within a screw: the exterior one, as usual, has its female in the end of the tripod, and the female of the interior screw is within the exterior; the interior one is longer than the other, its flat end rests on a small cup on the top of the support, and its milled head is a little above the other. Now by this arrangement we gain three distinct motions: for by turning both screws together, an effect is produced equal to the natural range of the exterior screw; by turning the interior one alone the effect produced is what is due to this screw; and by turning the exterior one alone (which may be done, because the friction of the interior screw in the cup is greater than that which exists between the two screws,) an effect is produced equal to the difference of the ranges of the two screws. Thus, were the exterior one to have 30 turns in an inch, and the interior 40, the effect last described will be exactly equal to what would be produced by a simple screw of 120 threads in an inch."

the air-bubble will steadily remain in the middle of the level during an entire revolution of the instrument in azimuth. These adjustments should be first performed approximately, for if the third foot-screw is much out of the level, it will be impossible to get the other two right. The vertical axis is now adjusted.

The next adjustment is to set the vertical circle at such a height that its two reading microscopes shall be directed to two opposite points in its horizontal diameter, which is done by raising or lowering the Y's which carry the horizontal axis.

The next adjustment is the levelling of the horizontal axis by means of the striding-level, the whole of which operation is in all respects the same as that described for levelling the transit axis, to which therefore the reader is referred. After performing this, the preceding adjustment must be examined, as it will probably be deranged. Indeed it is better first to set the axis horizontal, and then, by equally raising or depressing the two ends, to bring the miscroscopes into a diameter, and finally level again.

The adjustment for the line of collimation requires not only that the middle vertical wire shall describe a great circle, but that the middle horizontal wire shall have a definite position with respect to the divisions of the limb. It is usual to rectify the position of one of these at a time, taking the middle vertical wire first.* The error of this wire is ascertained and corrected, precisely in the same manner as that of the transit-instrument; with this difference, that, instead of taking the axis out of its bearings and turning it end for end, the whole instrument is turned half round in azimuth, which is an equivalent operation. The middle horizontal wire may be adjusted in the following manner: "Point the telescope to a very distant object, bisect it by the middle horizontal wire (near the intersection of the wires,) and read off by the microscopes the apparent zenith distance; now reverse the instrument in azimuth, and turning the telescope again upon the same object, bisect it as before, and again read off the angle which they show. One of these angles will be an altitude, and the other a zenith distance;" and, if there is no error, the sum of the two readings will be 90°, and half of what it differs from 90° will be the error of collimation, which may be either applied to correct any observation made during its existence, or removed in the following manner. One of the readings being the zenith distance, and the other the altitude of the object, reduce the zenith distance to an altitude, or *vice versâ*, and take the mean; it is evident that "the mean of the two will be the true zenith distance or altitude respectively; and while the telescope bisects the object, the microscopes must be ad-

* We speak of the middle wire only, as the side wires are supposed to be fixed parallel to it by the maker, and cannot be adjusted by the observer.

justed by their proper screws, so as to show that mean. This process may be repeated for obtaining a greater degree of accuracy; but its final determination should be deduced from observations upon many heavenly bodies, and the minute error that may remain unadjusted had better be allowed for." This and the preceding operation may be more conveniently performed by a collimating telescope.

The adjustment for setting the cross-wires truly vertical, is the same as that described as belonging to the portable transit; the position of the horizontal wires will then depend on the maker, or the horizontal wire may be put right by making it thread an equatorial star at its transit, when the vertical wires will depend upon the maker.

In conclusion it may be observed, that during a series of observations, if the instrument should be detected to be a small quantity out of level, (having previously gone through the principal adjustments,) it may generally be restored by means of the foot-screws only, when they require but a slight touch to effect it: this is more especially essential when the level of the horizontal axis is the one deranged, as correcting it by moving the Y's would derange the adjustment of the vertical circle with regard to its reading microscopes, the construction and adjustments of which it will next be necessary to describe.

The error of the vertical axis is to be detected by the hanging level, and can very readily be allowed for in computing the observation: as a general rule, when great accuracy is required, it is easier and safer to adjust by computation than by mechanical contrivances.

THE READING MICROSCOPE.

Fig. 1.

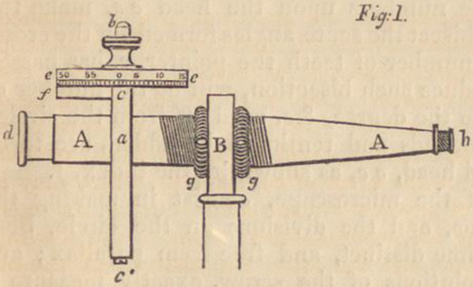
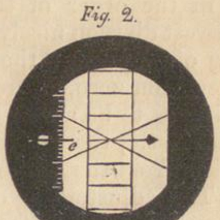


Fig. 2.



The divisions on the graduated circle indicate spaces of five minutes each, which are read off along with the *degree*, by means of an index-pointer. The remaining minutes and seconds, if any, are determined by the reading microscope, as was stated when describing the construction of the circle; it now remains to explain the principal parts of the micrometer, the method of adjusting it, and its application to practice. A A, fig. 1, represents the microscope, passing through a collar or support, B, where it is firmly held by the milled nuts, *g g*, acting on a screw cut on the tube of the microscope. These nuts also serve for placing the instrument at the proper distance, for distinct vision, from the divisions it is employed to read. In the body of the microscope, at *a*, the common focus of the object and eye-glasses, are placed two wires, crossing each other diagonally, and they are made to traverse the field of view either up or down, by turning the micrometer-screw, *b*, working in the box, *c c′*. Fig. 2 shows the field of view, with the magnified divisions on the instrument, as seen through the microscope. The shaded part represents the diaphragm, with the cross-wires, the angle made by which, may, by turning the micrometer-screw, *b*, be bisected by any line on the circle in the field of view, as is shown in the figure. On the left hand of the diaphragm appears the comb or scale of minutes, each of the teeth representing one minute. Movable with the wires along the comb is a small index or pointer, *e*, which in the figure is represented at zero, the centre of the scale, as is shown by its bisecting the small hole at the back of the comb. Now one revolution of the screw, *b*, moves the wires and the pointer over one tooth of the comb, that is, over a space equal to one minute; and part of a revolution moves them but a fraction of a minute. To determine this fractional quantity, a large cylindrical head, *e e*, is attached to the screw, having its edge divided into 60 equal parts, representing seconds, the index being fixed opposite the eye of the observer at *f*. In reading off an angle by this instrument, observe first the degree and nearest five minutes shown by the pointer on the graduated circle; then apply to the microscope, and by turning the screw, *b*, in the order of the numbers upon the head *e e*, make the nearest division nicely bisect the acute angles formed by the crossing of the wires; the number of teeth the pointer, *e*, has passed over from zero, to produce such bisection, will be the number of minutes to be added to the degree, &c. read off from the circle; and lastly, the odd seconds and tenths to be added, are to be taken from the divided head, *e e*, as shown by the index, *f*.

The adjustments of the microscope, consist in making the cross-wires in its focus, and the divisions on the circle, both appear at the same time distinct, and free from parallax; and also making five revolutions of the screw, exactly measure a five-minute space on the graduated circle. For the former of these adjustments, draw out the eye-piece, *d*, until distinct vision of the wires is obtained, and observe if the divisions of the instrument are also well defined, and whether any motion of the eye causes the least apparent displacement or parallax of the wires with respect to the graduations. If such a dancing motion be found, the microscope must be moved to or from the circle, by turning the nuts, *g g*, unscrewing one and screwing the other, until the wires and graduations both appear distinct, and no parallax can be detected.

Next, to examine and adjust the *run* (as it is termed) of the screw. If the run has been carefully adjusted by the maker, and no alteration made in the body of the microscope, the image of the space between two of the divisions will be exactly equivalent to five revolutions of the screw, when the wires and divisions are both seen distinctly. Let us, however, suppose that the length of the microscope has been deranged, and that the run is too great; for example, that the space of $5'$ on the limb is equal to $5'\ 10''$ by the micrometer, or that the image is too large. Now the magnitude of the image formed by the object-glass of the microscope, depends entirely on the distance of the object-glass from the limb, and is diminished (in the ordinary construction of the microscope) by increasing the distance between the limb and the object-glass, and *vice versâ*. In the case supposed, the image is too large, therefore the object-glass *h* must be removed further from the limb. Let this be done by turning the screw at *h* in or towards B. The image now will not be formed at *a*, as it ought to be, but nearer to B, and distinct vision must be gained by bringing the whole body of the microscope nearer to the limb. In this way, by two or three attempts cautiously conducted, we shall make five revolutions of the cross-wires correspond exactly with the image of the space between two divisions; and for greater accuracy the $5'$ should be read on each side zero, or $10'$ on the limb made equal to 10 revolutions of the micrometer.

The screw, c', gives motion to the comb or scale of minutes; and the micrometer-head being adjustable by friction, can be made to read either zero, or any required second, when the cross-wires bisect any particular division, by holding fast the milled-head, *b*, and at the same time turning the divided head, *e e*, round until its zero, or whatever division you require, coincides with the index, *f*: this, it will readily be perceived, is the means of accomplishing the adjustment spoken of at page 96.

Use of the Altitude and Azimuth Instrument.

This is the most generally useful of all instruments for measuring angles, being applicable to geoditical as well as astronomical purposes. In the hands of the surveyor it becomes a theodolite of rather large dimensions, measuring with great accuracy both vertical and horizontal angles. It does not possess the power of repetition; but the effect of any error of division on the azimuthal circle may be reduced or destroyed, by measuring the same angle upon different parts of the arc; thus,—After each observation, turn the whole instrument a small quantity on its stand, and adjusting it, again measure the required angle. A fresh set of divisions is thus brought into use at every observation, and the same operation being repeated many times, where great accuracy is required, the mean result may be considered as free from any error that may exist in the graduation. A repeating stand has, of late years, been frequently added to this instrument, and is a most powerful and convenient appendage, when great accuracy is required in the measurement of azimuthal angles. The two opposite micrometers being read off at each observation, will always remove the effect of any error in the centring. The vertical angles should, in all cases, be taken twice, reversing the instrument before taking the second observation, when (as before observed) one of the readings will be an altitude, and the other a zenith distance; the sum of the two readings, therefore, if the observation be made with accuracy, and no error exists in the adjustments or the instrument, will be exactly 90°; and whatever the sum differs from this quantity is double the error of the instrument in altitude, and half this double error is the correction to be applied + or — to either of the separate observations, to obtain the true altitude or zenith distance, + when the sum of the two readings is less than 90°, and — when greater.

In applying the instrument to astronomical purposes, it was formerly the custom to clamp it in the direction of the meridian, and after taking an observation, or series of observations, with the face of the instrument one way, to wait till the next night, or till opportunity permitted, and then take a corresponding series of observations of the same objects, with the face of the instrument in a reversed position. But this method being attended both with uncertainty and inconvenience, it is now usual to complete at once the set of observations, by taking the altitudes in both positions of the instrument as soon as possible after each other. When the meridian altitude is required, several observations may be taken, a short time both before and after the meridional passage; with the face of the instrument in one direction, and with it reversed, noting the time at each observation; and if we have the exact time of the object's transit, its hour angle in time, or its distance from the meridian at the moment of each observation, may be deduced. This, with the latitude of the place (approximately known) and the declination of the object, affords data for computing a quantity called the reduction to the meridian, which added to the mean of the observed altitudes, when the object is above the pole, and subtracted when the object is below the pole, will give the meridional altitude of the object, and *vice versâ* for zenith distances. The nearer the observations are taken to the meridian, the less will the results depend upon an accurate noting or knowledge of the time.

To compute the Reduction to the Meridian.

Practical Rule. Take from Table VII. the natural versed sines of the hour-angles, or times of each observation from the time of transit separately, and take their mean; then to the log. of this mean, add the log. co-sine of the assumed latitude, the co-sine of the declination, the co-secant of the meridian zenith distance, and the constant log. 9.31443; the sum, rejecting the tens from the index, will be the log. of the reduction in seconds of space.

The meridional zenith distance employed in the computation need only be approximate; if the latitude of the place and the declination of the body be nearly known, the meridional zenith distance will be equal to the difference between the latitude and the declination, when both are north, or both south, but equal to their sum, when one is north and the other south: and the meridian zenith distance of an object below the pole, is equal to the difference between 180°, and the sum of the latitude and declination.

As an example, we shall take that given in WOODHOUSE's Astronomy, vol. i. page 422, of the star Arcturus, as observed at the Dublin Observatory.

	Face of Inst.	Observed Alt.	Hour Angle in Time.	Versed Sine.
		° ' "	m s.	
East of meridian	E.	56 40 5,2	10 35 3	1067
	E.	42 22,9	6 35 3	0413
West of meridian	W.	45 10,0	2 47 7	0074
	W.	43 23,1	7 48 7	0580

Means..........	56 42 45,3	 533,5	log.	=	2·7271
Reduction .... +	1 52.4	° '			
	———	Latitude......53 23	cos.	=	9·7756
	56 44 37,7	Declination ...20 7	cos.	=	9·9727
Refraction	37,8	Mer. Z. D. = 33 16	cos.	=	0·2608
	———	Constant log.		=	9·3144
Meridian Alt.....	56 43 59,9	' " "			———
	———	Reduction .. 1 52,4 = 112,4	log.		2.0506

If the star be supposed known, the meridian altitude thus determined may be employed in correcting an assumed latitude; or, if the latitude be known, the star's declination may be obtained.

The latitude of a place is its distance from the equator, north or south, and it is equal to the elevation of the celestial pole above the horizon, or to an arc of the meridian contained between the zenith and the celestial equator; which arc can readily be determined, by observing the greatest or meridional altitude of a celestial object whose declination at the time is known; for when the declination is greater than the zenith distance, both being of the same denomination, (either both north or both south) the latitude will be equal the declination, *minus* the zenith distance. When the declination and zenith distance are of contrary denominations, then the declination *plus* the zenith distance will be the latitude. And lastly, when the zenith distance is greater than the declination, then the zenith distance *minus* the declination will be the latitude. And always of the same denomination as the greater of the two.

Another method of determining the latitude, is by observing the meridional zenith distance of a circumpolar star, both at its upper and lower culmination; then, computing the refraction for each observation, the co-latitude will be equal to half the sum of the two zenith distances added to half the sum of the two refractions. The latitude thus obtained does not depend on a previous knowledge of the declination of the object observed.

The method of determining the latitude by an observation of the altitude of the pole-star *at any time of the day*, together with the necessary tables, is given in the Nautical Almanac, (as newly arranged.)

A very successful and useful application of this instrument, is the determination of time and the direction of the true meridian, by equal altitudes and azimuths; the method of conducting a series of observations of this kind has been so clearly explained by the late Mr. Wollaston, in his *Fasciculus Astronomicus*, that we shall at once transcribe it nearly in the author's own words.

"In the morning, two or three or more hours before noon, let him (the observer) point the telescope toward the sun, and a little above it, and clamping the vertical circle, let him follow the sun till its upper limb just touches the first horizontal wire. Then, noting down the exact second of time, as shown by his chronometer, when that happened, let him follow the sun again till its upper limb just arrives at the second horizontal wire. After setting that down as before, let him prepare for the third or central wire; by now clamping the instrument in azimuth likewise, and holding its adjusting screw between his finger and thumb, let him bring the preceding limb of the sun just to touch the third or central perpendicular wire, at the same instant that the upper limb just touches the third or central horizontal one. Noting that instant, and setting it down, let him now read off the azimuth marked on the azimuth circle, and set it down under the other, and then prepare for making the preceding limb to touch the fourth perpendicular wire, at the same instant that the upper limb arrives at the fourth horizontal one; setting that time down again, and reading off the azimuth again, and setting it down, let him do the same by the fifth wire at each way, and record them as before. He will now find the lower limb of the sun, and its second or following limb, ready for observing in the same way, at the first, second, and third wire, making each perpendicular wire a tangent to the sun's last limb, at the instant that its lower limb just leaves the correspondent horizontal wire; and setting down the time, and after reading off the azimuth, setting that down too under the other. After these, the instrument may be released in azimuth, and the lower limb alone be observed, as it quits the fourth and fifth horizontal wires respectively.

"As soon as the sun has thus passed all his wires, he should read off at both the microscopes the zenith distance and altitude at which he had clamped the vertical circle; and if he has a barometer and thermometer, he should set down their station at the same time; for though he probably will have no occasion to regard the precise altitude at which he made these observations, yet if anything should deprive him of the correspondent ones, he may wish to have it in his power to deduce his time or his azimuth from them, and the reading off the microscopes after all is over is attended with very little trouble.

"These things will appear at first hurrying, and till a person becomes a little accustomed to it they certainly will be so. But after a little practice there will be found time enough to go through the whole with ease; for the vertical circle remains clamped the whole time, and all the six azimuths lie much within the limits of their adjusting screw.

"The easiest method of keeping so many observations from confusion, is to have a slate, or sheet of paper, ready ruled into five columns, to correspond with the five wires in the telescope as they occur in succession, in which to write down the observation belonging to each wire, whether that be time or azimuth; for if any cloud or accident should deprive him of any one or more of his observations, he will then at once see afterwards which of them is missing, when he comes to compare the two sets together.

"Leaving the instrument clamped for altitude, and covered entirely from the sun's rays, he must wait till it is at the same distance from noon in the evening to resume his task. For that, he must hold himself ready against the time comes; and previous to it, he will do well to re-examine the adjustment of his instrument, to be certain that no change has happened in the stand or the central cone, so as to throw its axis out of a perpendicular. Let him then observe the same method in this second set of observations as he did in those of the forenoon, considering those wires as first, at which the sun's limbs touch first, and setting down the times of their appulse to each respective horizontal wire, and bringing the preceding or subsequent limb to the corresponding perpendicular one, and reading off the azimuths just as he did before. When all are passed, he may release all the clamps, and replacing his shade, leave the instrument till he has reduced his observations of corresponding altitudes: if he has observed them all, he will have obtained ten pair, and of azimuths six pair, which he must now select from each other, and properly class them, by taking the last in the morning, in conjunction with the first in the evening, and so on, till each observation is paired with its opposite corresponding one."

The time of the meridional passage of the sun's centre, as indicated by the time-keeper employed, will be very nearly equal to half the sum of the times at which each pair of the observations were made, and would be exactly so if the declination did not change during the interval elapsed; (similar observations being made upon any star, the result will show the exact sidereal time of its transit.) The correction to be applied to the time of the sun's transit or apparent noon deduced as above, on account of the change of declination, may be computed by the following formula: *

$$\text{Make}\ \frac{T}{1440\ \sin.\ \frac{T}{2}} = -A$$

$$\frac{T}{1440\ \tan.\ \frac{T}{2}} = B$$

$$\text{correction} = \mp A\ .\ \delta\ .\ \tan.\ L + B\ .\ \delta\ .\ \tan.\ D$$

in which L = the latitude of the place (*minus* when south)

δ = the double daily variation in the sun's declination (deduced from the noon of the preceding day, to the noon of the following day; *minus* when the sun is receding from the north)

D = the declination, at the time of noon, on the given day (*minus* when south)

T = the interval of time between the observations expressed in hours.

Note. B is to be considered *plus*, when the interval of time is less than 12 hours, otherwise *minus*.

Practical Rule. To the constant log. 3·1584 add the log. sine of half the interval of time between the observations reduced

* Tables of equation of equal altitudes are contained in Mr. Baily's volume of Astronomical Tables and Formulæ, and in Schumacher's Hülfstafeln. The log. of double the sun's daily variation in declination, is given in the Berlin Ephemeris as log. μ, in the page relating to *true* noon.

to space, and subtract their sum from the log. of the whole interval, *expressed in hours and decimals;* call the remainder A, always *minus*.

To the constant log. 3·1584, add the log. tangent of the above half interval, and subtract their sum from the log. of the whole interval as before, and call the remainder B, *plus*, when the interval is less than twelve hours.

To A, add the log. of double the daily variation of the sun's declination, expressed in seconds, (*minus* when the sun is *receding* from the north,) and the log. tangent of the latitude, (*minus* when south,) the natural number corresponding to the sum to be considered as seconds of time, &c., *plus* or *minus* as it may result.

To B, add the log. of double the daily variation of the sun's declination, as before, and the log. tangent of the sun's declination, *minus* when south, for noon of the given day, the natural number corresponding to the sum, must be taken as seconds of time with its proper sign. The algebraic sum of these two quantities will be the correction required, and must be added to, or subtracted from, the half sum of the times of observation, according as it is *plus* or *minus*, to obtain the correct apparent time.

EXAMPLE.

(*From* Mr. BAILY's *Volume of Astronomical Tables, &c.*, p. 227.)

On July 25, 1823, in N. Lat. 54° 20′ at $8^h\ 59^m\ 4^s$ A.M., and at $3^h\ 0^m\ 40^s$ P.M. the sun had equal altitudes. Required the equation, or correction to be applied to the mean of those times, in order to find the time of noon. The interval of time is $6^h\ 1^m\ 36^s$, which converted into arc = 90° 24′, and by the Nautical Almanac the declination of the sun, at noon on that day, was + 19° 48′ 29″, and its double daily variation equal to — 25′ 29″ = — 1529″. The operation will therefore stand thus:—

Constant log. . =	3·1584	. . . . .		3·1584
$\frac{T}{2}$ = 45° 12′ sin. =	9·8510	. .	tangent =	0·0030
Sums . . . =	— 3·0094	. . . . .	=	— 3·1614
T = 6·0266 log. =	0·7801	. . . . .		0·7801
A = (Differences) =	— 7·7707	. . . .	B =	+ 7·6187
δ = — 1529″ log. =	— 3·1844	. . . . .	log.	— 3·1844
L = + 54° 20′ tan.	+ 0·1441	D = 19° 48′	tan.	+ 9·5563
+ 12^s,57 =	+ 1·0992	. .	— 2^s,29 =	— 0·3594

correction = + 12^s,57 — 2^s,29 = + 10^s,28

This value being added to the mean of the times of the observed altitudes, or $\frac{1}{2}$ ($20^h\ 59^m\ 4^s + 27^h\ 0^m\ 40^s$) $= 23^h\ 59^m\ 52^s$, will give $0^h\ 0^m\ 2^s,28$ for the time at apparent noon, to which, if the equation of time be applied, the result will be the time of mean noon.

The equal azimuths may similarly be employed for finding the direction of the true meridian. They must be opposed to each other in pairs, just in the same manner as corresponding altitudes; the first in the morning to the last in the evening, and so of the rest. Then deducting the one from the other, and applying half the difference between the two to the smallest number in each pair, it will give a number of degrees, minutes, and seconds, in which, if all the observations were perfect, the whole six pair would coincide; and if they do not, the fair mean deduced from among them will approach nearly to the truth, *i. e.*, the error of 180° on the azimuth circle from the true meridian.

To that mean point, deduced from these observations, the instrument must now be turned, and fixed there till the proper correction can be applied to it. Upon the telescope being turned down to the horizon each way, it may be observed what distinct object there may be, either to the north or south, that coincides with one of the perpendicular wires; or if no such object should occur, a mark may be placed each way, or either way, to which the instrument may be kept, till the correction can be investigated, which is requisite, on account of the change of the sun's declination during the interval between the morning and evening observations; for any alteration in his declination will affect the azimuth deduced in this way, as it does the hour. This correction is greatest about the time of the equinoxes, as the change in the sun's declination is then the most rapid: it may be computed from the following formula; but when deduced from a star, no such correction is requisite.

$$\text{Correction} = \tfrac{1}{2}\,(D' - D)\ \text{sec. Lat. cosec.}\frac{(T' - T)}{2}$$

In which expression, $(D' - D)$ = the change in the sun's declination during the interval between the observations, and $(T' - T)$ = the interval itself.

Practical Rule. To the log. of half the change of declination, add the log. secant of the latitude, and the log. co-secant of half the interval of time converted into space: the sum — 20 will be the log. of the correction in seconds of space.

When the sun is advancing towards the elevated pole, the middle point, or meridian, as found by equal altitudes, will be too much to the west of the true meridian, by the amount of this correction, and *vice versâ*, when he is receding from the elevated pole; therefore, the telescope being shifted in azimuth by the quantity thus computed, will be correctly in the meridian.

EXAMPLE.

On February 28, 1834. When the sun had equal altitudes, the azimuth circle read 130° 10′ 15″ and 32° 36′ 15″, therefore the middle point or reading of the approximate meridian was 81° 23′ 15″. The interval of time between the observations was 5 hours, the half of which converted into arc = 37° 30′ 0″. The sun's hourly change of declination = 56″,77, therefore the change for half the interval = 141″,92 (approaching the north pole.) The latitude of the place 51° 28′ 39″, required the correction to be applied to the middle point to obtain the direction of the true meridian.

$\frac{1}{2}(D' - D) = 141'',92$ ......log. $= 2{\cdot}1520436$
Lat. $= 51°\ 28'\ 39''$ ....sec. $= 0{\cdot}2056388$
$\frac{(T' - T)}{2} = 37°\ 30',0$ ....co-sec. $= 0{\cdot}2155529$

Correction $= 374'',31$ ....log. $= 2{\cdot}5732353$
$= 6'\ 14'',31$

		°	′	″
Reading of the middle point	=	81	23	15
Correction	—		6	14,31
Reading of the instrument when set to the true meridian	=	81	17	0,69

Another, and an easy method of finding a meridian line, where dependence can be placed upon the time shown by a chronometer (or watch) is to compute the time of the meridional passage of a star near the pole, either above or below the pole, and pointing the telescope of the instrument to the star, bisect it at the exact moment; when, if the adjustments of the instrument are perfect, the telescope will be very nearly in the plane of the meridian.

A third method, which admits of great accuracy, when instruments of large dimensions are employed, consists in bisecting a circumpolar star when at its greatest eastern and western elongations; a line bisecting the horizontal interval, contained between the two positions of the telescope, will be the direction of the meridian; this interval being measured on the azimuthal circle, and the telescope moved through half that interval, from either its eastern or western position, will place it in the meridian. But it will often be inconvenient to wait till the star attains its second greatest elongation; and as one of the observations must be made in the day-time, (except at particular seasons of the year,) a star will not be visible through telescopes of small size. To make a single observation available for the purpose, the azimuth (east or west) of a star, when at its greatest elongation, as well as the time of its attaining such position, must be computed (which may be done by the annexed rules), when the observer must first bisect the star, and follow it in its slow motion, until he is satisfied that it is stationary; or, what is perhaps better, if he is certain of his time, bisect it at the exact moment. The azimuth circle must now be read off, and the position of some fixed object, with respect to the azimuth of the star, should be determined; a lamp may at the time be placed at some distance for reference, and its azimuth being thus obtained, other objects may be referred to it at leisure.

To compute the azimuth of a circumpolar star, when at its greatest elongation.

Rule. From the log. sine of the polar distance, subtract the log. co-sine of the latitude: the remainder will be the log. sine of the azimuth required.

To compute the time (before or after its meridional passage) of a circumpolar star attaining its greatest elongation, either east or west.

Rule. Add together the log. tangent of the polar distance, and the log. tangent of the latitude: their sum, rejecting ten from the index, will be the log. co-sine of the hour-angle (in space); which, divided by fifteen, will be the sidereal time a star attains its greatest elongation before or after it passes the meridian at its upper culmination; therefore, having the time of the meridional passage (computed, as explained at page 82) the time of its greatest elongation will be known.

The star *α* Ursæ Minoris, commonly known as the pole-star, is well situated for determining the direction of the meridian by the above method: its apparent motion when near its greatest elongation is so small, that it appears stationary at that point for a considerable time, affording us an opportunity of observing it both by direct vision, and also by reflection; an advantage particularly great, as we need not depend upon the spirit-bubble in levelling the instrument, for the observations expose the slightest deviation, and enables us to correct its position; thus:—

Suppose the pole-star, by previous calculation, is ascertained to be at its greatest elongation at a certain time; having set up the instrument approximately level, place an artificial horizon in a proper position to observe the star by reflection: then direct the telescope to the star, and having bisected it with the intersection of the cross-wires, clamp the horizontal circle; now depress the telescope till you see the reflected image of the star in the artificial horizon, which if the instrument is perfectly level, will also appear bisected; if it does not, you must immediately correct *half* the *deviation* by the foot-screws of the instrument, which will set the instrument perfectly level. Bisect the reflected image of the star by giving motion to the horizontal circle; then carefully elevate the telescope to the star itself, which will also be bisected if the estimation of half the amount of deviation has been correctly made, and if not, it will be a nearer approximation, which must be perfected by a similar process, that is, by removing half the error with the foot-screws, and the other half by the horizontal circle.

Having now set the instrument, so that, upon elevating and depressing the telescope, both the direct and reflected images of the star appear bisected, a satisfactory observation will have been made. This being done at both the eastern and western elongations, and the readings of the azimuth circle noted, the middle point between the two readings will lie in the plane of the true meridian. Or, as before observed, one observation may be made available for the same purpose, by likewise observing a fixed object, as a lamp or church tower, and computing, by the foregoing rule, the azimuth of the star at that time, for the horizontal angle between the star and the fixed object, plus or minus the computed azimuth of the star (plus when the object is on the same side of the meridian, and further from it than the star; and minus, when it is nearer the meridian than the star, or when they are on opposite sides of the meridian) will give the azimuth of the fixed object from the north, from which the direction of the meridian may be found at any time.

It is only stars whose polar distance is less than the co-latitude of the place of observation, that can be used in the two latter methods of determining the direction of the meridian.

The last method which we shall advert to, and which is mostly applied to objects south of the zenith, consists in computing the azimuth of a celestial object, from an observation of its altitude, the latitude being known; at the same time observing the horizontal angle contained between it and any fixed object; for the difference or sum of the azimuth of the celestial body, and this observed horizontal angle, will be the angular distance of the fixed object from the meridian: the sum when the fixed object is on the same side of, and further from, the meridian than the celestial object, otherwise, the difference.

Formula for computing the azimuth of a celestial object from its observed altitude, &c.

$$\text{Tang. } \tfrac{1}{2} \text{ azimuth} = \sqrt{\frac{\sin.\frac{s}{2} - z.\ \sin.\frac{s}{2} - \lambda}{\sin.\frac{s}{2}.\ \sin.\frac{s}{2} - \pi}}$$

In which $\frac{s}{2}$ = half the sum of the polar distance, the co-latitude and the zenith distance, π and λ = the polar distance and co-latitude Z = the zenith distance of the object.

Practical Rule. Add together the polar distance, the co-latitude, and the zenith distance, and call their sum S. To the log. sine of half S *minus* the zenith distance, add the log. sine of half S *minus* the co-latitude, and increasing the index by 20, call the sum of the two logs. A.

To the log. sine of half S add the log. sine of half S *minus* the polar distance, and call the sum of the two logs. B.

From A subtract B, and divide the remainder by 2; the quotient will be the log. tangent of half the object's azimuth, which doubled will be the whole azimuth, or horizontal angular distance from the south meridian.

EXAMPLE.

On February 20, 1834, in latitude 51° 28′ 39″. The zenith distance of α Geminorum (east of the meridian) corrected for refraction = 56° 20′ 10″, the azimuth circle reading 125° 18′ 24″: after which the clamps of the instrument were released, and a fixed terrestrial object bisected, also to the *east*, but nearer the meridian than the star, the azimuth circle now read 83° 15′ 20″, consequently the horizontal angle between the star and the object = 42° 3′ 4″ required the azimuth of the object from the meridian.

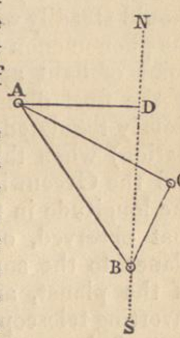
	° ′ ″	
π (from the N.A.)	= 57 45 16	
λ	= 38 31 21	
z	= 56 20 10	
	2) 152 36 47	
$\frac{s}{2}$	= 76 18 23	
$\frac{s}{2} - z$ . . .	= 19 58 13	sine = 9·5334322
$\frac{s}{2} - \lambda$ . . .	= 37 47 2	sine = 9·7872371
		A = 9·3206693
$\frac{s}{2}$	= 76 18 23	sine = 9·9874766
$\frac{s}{2} - \pi$ . . .	= 18 33 7	sine = 9·5026514
		B = 9·4901280
		A = 9·3206693
		2) 9·8305413 = A — B

°	′	″	
39	26	45·5	½ star's azimuth, tan. = 9·9152706
		2	
78	53	31·0	= azimuth of star
42	3	4·0	object nearer meridian
36	50	27·0	= azimuth of object east of south.

The verification of the meridional position of an instrument by observing the passage of a circumpolar star at both its upper and lower culminations, as well as the method by high and low stars, has been fully explained, when speaking of the transit; and as the altitude and azimuth circle, when firmly clamped in the plane of the meridian, becomes a complete transit instrument, and may be employed precisely in the same manner and for the same purposes, we refer for this use of it to the account which we have given of that instrument.

In addition to the method of determining differences of longitude by the observed transits of the moon and moon-culminating stars, (page 88,) we subjoin the following as applicable to the use of the instrument which we are now describing. The latitudes and longitudes of a great number of the most conspicuous places in this country, as church steeples, &c., having been determined, and published in the account of the Ordnance Survey, they afford a ready means of finding both the latitude and longitude of places adjacent to them, by means of trigonometrical measurement. The process may be understood from the following example.

Let A represent a place, the longitude and latitude of which are known; B the station, the situation of which we wish to determine; C any point to form the triangle; N S the direction of the meridian.



First, the angles at the three points must be observed, and one of the sides measured, when the distance A B must be computed by plane trigonometry. Suppose it to be = 6040·6 feet. Then the azimuth of A, from the meridian, or the angle, A B N, must be determined, which may be done by any of the methods we have described; suppose it = 56° 58′ 40″; now the line A D perpendicular to the meridian, and B D the difference of latitude of B and A, may be computed from the right-angled triangle A B D, having A B 6040·6 feet, and the angle A B D = 56° 58′ 40″; A D comes out = 5064·8 feet, and B D = 3292·2 feet.

With the latitude of A, which suppose = 51° 27′ 44″, enter Table VIII. and take out the length of a second, both of latitude and longitude; divide the distances A D and B D by those numbers, and the quotients will be the difference of longitude and latitude (in arc) required. Thus:

$$\frac{\text{A D} = 5064{\cdot}8}{\text{Table} = 63{\cdot}31} = 80{,}44'' = 1'\ 20{,}00'' = \text{diff. of long. in space.}$$
$$= 5^s{,}33 \text{ in time.}$$

$$\frac{\text{B D} = 3292{\cdot}2}{\text{Table} = 102{\cdot}02} = 32{,}27'' \text{ difference of latitude.}$$

		M.	S.	
Longitude of A	=	21	10,40	West.
Difference			5,33	East.
Longitude of B	=	21	5,07	West.

		°	′	″	
Latitude of A	=	51	27	44,00	North.
Difference......			0	32,27	South.
Latitude of B	=	51	27	11,73	North.

Lastly, we shall give the method of finding the longitude by observations of the eclipses of Jupiter's satellites.

The Nautical Almanac contains the Greenwich mean time when the phenomena happen, consequently the estimated longitude of the place being applied to the time therein given, will be the time at which an eclipse may be expected to happen at the station of the observer, who, being at his telescope a few minutes before, should steadily watch the spot near the body of the planet where the phenomenon is expected, till he discovers the first glimpse or point of light appear, if it be an emersion from the shadow, or of the final disappearance, if an immersion; noting, by a previously regulated time-keeper, the exact mean time (at his own station) when this happens. The difference between this time and the Greenwich time given in the Nautical Almanac, will be the longitude in time; east, if the Greenwich time is less than that observed, otherwise west. Before the opposition of the planet to the sun, the eclipses always happen on the west side of the planet, and afterwards on the east. But when using an inverting telescope, the appearance will be reversed. The situation of the satellite with respect to the planet where the eclipse takes place, is given in the Nautical Almanac.