The mathematical principles of mechanical philosophy, and their application to the theory of universal gravitation

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Preface.

PREFACE.

A LEADING object that I have had in view in preparing the present Treatise has been to gather into one uniform system the principles of mechanical science, beginning with the most elementary and ascending to the most general. In attempting to accomplish this I have collected the fundamental principles into separate Chapters, and placed after them Chapters of application of these principles to the demonstration of others of a second class, and have then added collections of problems and, in some instances, hints to guide to their solution.

An attachment, and that in most respects a laudable attachment, to the geometry of the *Principia* had, till of late years, led to the practice of retaining in our course of University reading some parts of that immortal work, rather for the beauty and elegance of its demonstrations, than for the importance of the theorems demonstrated. But this practice has been gradually sinking into disuse, a result which we owe to Professor Woodhouse's *Physical Astronomy*, to M. Poisson's *Traité de Mécanique*, which has been extensively used amongst us, and very largely to Mr Whewell's Treatises on *Statics* and *Dynamics* and

Mr Airy's *Mathematical Tracts*. But notwithstanding the great and happy changes thus brought about we still cling to the old methods, not as a whole, but just so far as to derange our system and give to it the ambiguous character of being neither strictly geometrical, nor strictly analytical. But I wish not to be misunderstood; I mean not to imply that geometry should be discarded and banished from our academical course of study; far from it; for the analyst will find his analysis of little benefit if he have not the power of gathering from his formulæ geometrical conceptions. Neither would I have it for a moment conceived, that I would in the least degree repudiate the profound veneration, which is so justly due even to the letter of the *Principia:* my own admiration of the clearness and conciseness of its demonstrations rather induces me to invite others to participate of the pleasures they may enjoy from its attentive and diligent study. But this I desire, that we should pay more regard to system than we hitherto have done; if our course is to be geometrical let us adhere to geometry, if analytical to analysis; if we are to admit both (the preferable course) let us keep our systems well apart, and not have our course of reading confused, here analysis and there geometry.

My own experience has impressed me also with the conviction, that many of our candidates for University honours are debarred the high enjoyment of penetrating into the sublimer investigations of Physical Astronomy from the want of some treatise that would lead them by a clear and distinct path, and with an undivided attention, through the train of reasoning which leads from elementary mechanical principles to the demonstration of celestial phenomena. Some, it is true, of our first rate students do attain this eminence; but might not this few be considerably augmented, if their path were well pointed out and disencumbered of many of the obstacles which lie in their way and impede their course?

Let it not be imagined, however, that I send forth the present volume with the presumptuous confidence, that the want of a complete analytical system of mechanics is supplied by its appearance; though I will so far commit myself as to confess, that to supply this want has been my earnest desire;—no, I would rather use the experience of a distinguished Author, whose name I have already used, who is a far better judge, in such a case, than myself, and say in his words, "a few years experience has a great tendency to diminish the confidence of producing what shall satisfy himself and others, with which a young author sets out: and he learns that the vivid impression of fancied deficiencies and imperfections of preceding works which at first induced him to write, is a very insufficient warrant of his own skill and judgment." But yet my object has been unique; and it has not been till after much time and thought spent upon the subject, that I have ventured to lay my work before the public: how I have satisfied my own desire I leave to the candour of my readers to determine.

In the first, second, and third Chapters of Statics will be found the principles of the composition and equilibrium of statical forces acting, first on a particle, then on a rigid body, and lastly on a system of bodies connected in any manner. From the conditions of equilibrium the principle of Virtual Velocities has been deduced. Afterwards this principle has been demonstrated independently after the method of Lagrange, and from it are deduced the conditions of equilibrium. Then follow Chapters of application. The fourth Chapter is a collection of examples of finding the centre of gravity of bodies, being an application of the formulæ for the co-ordinates of the centre of parallel forces. In this Chapter I have aimed at explaining and illustrating integration between limits: see particularly Examples 8, 13, 14, 19, 25, 26. The fifth Chapter contains the application of the principles to the six mechanical powers, and concludes with some notices of the laws of friction. The sixth Chapter is upon roofs, arches, and bridges, which form interesting applications of the principles of equilibrium. In this Chapter will be found some remarks upon the roofs of Trinity College Hall and Westminster Hall, the action of buttresses, and the stone vaulting of King's College Chapel, as well as notices of other noted edifices and structures. An example is given of the method of calculating the lengths and weights of the supporting rods, chains, and road-way of suspension bridges, that the strain may in every part be proportional to the strength of the chain. Then follows a Chapter of statical problems, beginning with some general remarks on their solution. And the treatise on Statics closes with a Chapter on Attractions. After calculating the attraction of spherical and spheroidal bodies of homogeneous mass I have proceeded to the more general investigation of the attraction of a body differing but little from a sphere in form, with a view to the calculation of the Figure of the Earth in a future part of the work. This has led me to introduce Laplace's Coefficients, a subject unknown in our University course, till introduced a few years since by Mr Murphy in his Treatise on Electricity. I have followed Laplace's course, and not the inverse method of Mr Murphy. The frequent occurrence of the equation

$$\frac{d^2 V}{dx^2} + \frac{d^2 V}{dy^2} + \frac{d^2 V}{dz^2} = 0$$

in physical investigations makes it highly desirable, that a knowledge of the profound analysis of Laplace should be made as familiar as possible to the higher class of students in the University. For this reason I have introduced, in as concise and at the same time as clear a manner as I was able, the principal properties of the Coefficients of that great analyst, breaking up and arranging the subject in the form of propositions.

The treatise on Dynamics opens with a Chapter upon the fundamental principles of the motion of bodies, which I gather wholly from experiment and observation. After explaining the conventional method of measuring motion I proceed to an enquiry into the laws that regulate the motion of bodies when uninfluenced by external causes: a variety of experiments and facts of ordinary occurrence point to the principle called the First Law of Motion. This leads to an explanation of the conventional method of measuring force dynamically. An investigation is then made into the laws which regulate the motion of bodies when acted on simultaneously by different causes, and this leads to the principle called the Second Law of Motion. This law enables us to introduce a method of referring the motion of a body to three rectangular axes. The necessity is then shewn of obtaining a relation between the two arbitrarily assumed measures of force, viz. pressure, the statical, and velocity generated, the dynamical measure: this leads to the principle called the Third Law of Motion. The introductory Chapter of Dynamics concludes with the enunciation of a self-evident principle, analogous to that first introduced by D'Alembert, whereby we deduce the equations for calculating the motion of a system from the equations of equilibrium. This principle is, in fact, the interpretation of the Three Laws of Motion into analytical language. In explaining the means of estimating force I have aimed at giving a distinct idea of the nature of forces that require a finite time to develop their effects, and those which generate velocity in an indefinitely short time.

The second Chapter is upon the motion of a single particle. In this I have entered fully into the properties of central forces, and calculated the motion in various cases: and at the close of the Chapter Kepler's Laws are made use of to guide us to the discovery of the nature of the force acting on the planets; and we thus catch a first glimpse of the theory of gravitation: after shewing that there is sufficient ground to justify us in undertaking the task of calculating the consequences of this law, a large portion of the remainder of the work is devoted to that enquiry. In the third Chapter the motion of two particles attracting each other according to the law of gravitation is calculated. In the fourth the perturbations in this motion by the introduction of a third attracting body are explained upon the supposition that the disturbing body is very distant. This Chapter contains the Sixty-sixth Proposition of the First Book of the *Principia* and its corollaries and some Propositions of the Third Book put into an algebraical form: this is a digression from the chain of exact reasoning which is the professed object of the work; but being in a separate Chapter the student, if he choose, may pass over this and proceed to the fifth Chapter, in which the distance, longitude, and latitude of the Moon are calculated to a second approximation upon the theory of gravity. The sixth Chapter contains the calculation of the perturbations of the planets. In the Lunar Theory the only part peculiar to this work is the way in which I have introduced the constants c and g, which give rise to the motion of the line of apsides and the line of nodes. In the Planetary Theory I have used M. Pontecoulant's method of integrating the equations of motion of an undisturbed planet and then applied Lagrange's principle of the variation of parameters to calculate the variations of the elliptic elements. This Chapter closes with a demonstration of the Stability of the Planetary System, retaining the squares of the eccentricities and inclinations. The next is upon the motion of a particle on curves and surfaces and also on the oscillations and perturbations of pendulums. A collection of problems on the motion of bodies considered as particles follows.

The next two Chapters contain the calculation of the motion of a rigid body acted on, first, by forces of finite intensity, and secondly, by instantaneous or impulsive forces. The subject of the first of these Chapters leads to the demonstration and calculation of the Precession of the Equinoxes and the Nutation of the Earth's Axis: the formulæ for these are prepared for numerical calculation; the reduction to numbers will be found at the close of the Chapter on the Figure of the Earth. After calculating the motion of a flexible body in the simple case of a vibrating cord, I close the treatise on Dynamics with the fifteenth and sixteenth Chapters, in which are developed the General Dynamical Principles of the Motion of a Material System, and their application to the solution of dynamical problems, of which a large collection is given.

In the treatises on Hydrostatics and Hydrodynamics the general principles of those Sciences are developed, and applied to the determination of the Figure of the Earth upon the hypothesis of its mass having been at some former epoch in a semi-fluid state, the form of the atmospheres of the planets, the tides, and the effect of a resisting medium upon the elements of the planetary orbits.

Lastly, a summary of the arguments in favour of the Theory of Universal Gravitation closes the Work. A reference to the Table of Contents will give a better view of the character of the Work than I have deemed it necessary to give in this place.

The prevailing argument with me for using the old differential and integral notation is the excellence of Fourier's notation for definite integrals: I much prefer that to any other that I have seen, and this naturally led me back to the old form of differentials and integrals. In case any of my readers are not acquainted with Fourier's notation I now give it, $\int_a^b u dx$ represents the integral of the differential coefficient u, or of the differential udx, with respect to x taken between the limiting values a and b of x. In successive integration the order of arrangement of the integrals is the same as that of the differentials: thus $\int_{-1}^{1}\int_{0}^{2\pi} P d\mu d\omega$ represents the double integral of P with respect to μ and ω, the limits of μ being -1 and 1, and the limits of ω being 0 and 2π.

I repeat, that it is not with the expectation that I have fully succeeded in satisfying even my own desires that I present this volume to the students of the University, but with the earnest wish that it may be found useful and that my labours may not have been altogether spent in vain. Should any of my readers favour me with any suggestions of improvement I shall receive them with the greatest thankfulness.

J. H. P.

CAIUS COLLEGE,
Nov. 26, 1836.

ERRATA.

PAGE	LINE	ERROR.	CORRECTION.
20	11	Q	R
65	6 from bottom	y	$\bar{y}$
166	last line	$\}$	$\}^{\frac{1}{2}}$
169	6 from bottom	R_i	$R_{i,}$
251	3	ε	ϵ
312	2 from bottom	$=$	$-$
488	3, 6	$\Sigma . m P dr$	$\Sigma . P dr$
528	3	DIFINITIONS	DEFINITIONS
528	7 from bottom	*incomprehensible*	*incompressible.*
562	18	*afford*	*affords.*