An account of the life, writings, and inventions of John Napier of Merchiston

Stewart, David

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Section II. Napier's bones.

Such was the ſtate of arithmetical computation, at the time of the invention of the Logarithms, which, as Napier himſelf ſays, *Omnem illam priſtinæ matheſeos difficultatem penitas e medio tollit; et ad ſublevandam memoriæ imbecillitatem ita ſe accomodat, ut illius adminiculo facile ſit, plures quæſtiones mathematicas unius horæ ſpatio, quam priſtinia et communiter recepta forma ſinuum, tangentium et ſecantium, vel integro die abſolvere* *. But before we proceed to this moſt important diſcovery, we ſhall give an account of thoſe ingenious contrivances, intended to anſwer the ſame purpoſe, which previouſly occurred to Napier.

SECTION II.

NAPIER'S BONES.

THE firſt of theſe mechanical devices is what our author calls Rabdologia, or the art of computing by figured rods. Theſe rods are ſquare parallelepipeds three inches in length, and three tenths in breadth. Each of the faces of theſe parallelepipeds is divided into ten equal parts, of which nine are ſquares and in the middle, and half of the tenth at one extremity or the top, and half at the other extremity or the bottom. Every one of theſe ſquares is cut by a diagonal from left to right upwards. At the top of each face is ſome one of the ten digits 0, 1, 2, 3, &c.

In

* Log. Canon. deſcriptio. in dedic.

In the firſt ſquare below that digit is repeated, in the ſecond is its double, in the third it's triple, and ſo on. Of theſe multiples of the digit, the figure of units is below, and the figure of tens above the diagonal. The meaning of what has been juſt ſaid will be evident by a little attention to Fig. I. where the four faces of each rod of the ſet, recommended by Napier, are unfolded. By means of theſe rods the operations of multiplication and diviſion are performed by addition and ſubtraction.

The rule for multiplication is—Bring the rods to form the multiplicand at the top of their upper face. Join a rod, having unity at the top of its upper face, to the right or left hand ſide; in which ſeek the right hand figure of the multiplicator, and write out the numbers correſponding thereto in the ſquare of the other faces, by adding the ſeveral numbers occurring in the ſame rhomboid. Seek the ſecond figure of the multiplicator and proceed in the ſame manner: arrange and add the numbers wrote out, as in common multiplication; the ſum is the product required. To multiply 1785 by 364, for example, I diſpoſe the proper rods as in Fig. II.; next to 4 (the firſt right hand figure of the multiplicator) I find 0; in the contiguous rhomboid 2 and 2, which added together make 4; in the next 3 and 8 which make 1 and a ſurplus of ten; and in the laſt 2 and 4 which, together with unity for the ten I had in the former rhomboid, make 7. Theſe numbers 0, 4, 1, 7, I ſet down one after the other as I find them, proceeding from right to left. I go on in the ſame manner with 6 and 3 (the other figures of the multiplicator); and, after arranging and adding the partial products I find the total product required. Thus,

364

364
7140
10710
5355
649740

The rule for division. BRING the rods to form the divifor at the top of their upper face. Join a rod having unity at the top of its upper face, to the right or left hand fide. Defcend under the divifor till you meet thofe figures of the dividend wherein it is firft required how often the divifor is found, or the next lefs number, which fubtract from the firft figures of the dividend, and put for the firft figure of the quotient the correfponding number on the fide face. Bring down, one after the other, the remaining figures of your dividend as in common divifion, and proceed in the fame manner till you have finifhed the operation. Let it, for example, be required to divide 649740 by 364. I difpofe the rods as in Fig. III. The next lefs number under the divifor 364 to 649 (the firft figures of the dividend) I find to be 364 itfelf which I fubtract from 649 putting 1, the number correfponding on the fide face, for the firft figure of my quotient: to my remainder 285 I bring down 7 the next figure of my dividend. The next lefs number to 2857 under the divifor I find to be 2548, which I fubtract from 2857, putting 7, the number correfponding in the fide face, for the fecond figure of the quotient. I go on in the fame manner till I have brought down the other figures of the dividend and completed my quotient as follows.

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649740(1785
364
———
2857
2548
———
 3094
 2912
———
 1820
 1820
———
  (0)
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Although the extraction of the ſquare and cube roots may be pretty expeditiouſly performed by the rods, Napier propoſes an auxiliary lamella for the abridgement of it. It would ſerve little purpoſe to give a particular deſcription of the lamella, or an account of the manner of uſing it. Its length and thickneſs are the ſame with thoſe of the rods, and its breadth quadruple. Its two faces are divided and marked as in Fig. IV. To find out the way of operating by it will be no difficulty to any body who is a little acquainted with arithmetic and has time to ſpare.

Another of Napier's contrivances is his *multiplicationis promptuarium.*

This machine conſiſts of a box of figured lamellæ. The lamellæ, two hundred in number, are each eleven inches in length and one inch in breadth. Each of theſe lamellæ is divided into eleven equal parts of which ten in the middle are ſquares, and two thirds of the eleventh at one

one extremity, and one third at the other. Every one of thefe fquares is divided into nine lefs fquares, one hundred of the lamellæ are each one fourth of an inch in thicknefs, and the other hundred one eighth. Suppofe the former, which we fhall call direct lamellæ, placed fo that the greater margin may appear at top and the lefs at bottom; and the latter which we fhall call tranfverfe, placed laterally, with the greater margin to the right and the lefs to the left hand. In this pofition every fquare appears cut by a diagonal (faint in the fmall but ftrong in the great ones) from the left to right upwards. Each of every ten both of the direct and of the tranfverfe lamellæ has fome one of the ten digits 0, 1, 2, 3, &c. infcribed on its greater margin. The multiples of the digit on the margin of a direct lamella are difpofed in each of its greater fquares as pointed out by Fig. V. where *a* reprefents the digit itfelf, *b* the right hand figure, and *b′* the left hand figure of its double; *c* and *c′* the right and left hand figures of its triple (the plain letters being above and the accented ones below the diagonal of the figure); *d* and *d′* thofe of its quadruple, and fo on. In the tranfverfe lamellæ thofe which have *o* on the margin are untouched; thofe which have unity on the margin have the loculus correfponding to *a* cut out; thofe which have two on the margin have the loculi of 6 and 6′ cut out; thofe which have 3 the loculi of *c* and *c′*; thofe which have 4 the loculi of *d* and *d′*, &c. This will be fufficiently evident by infpecting Fig. VI. where it is examplified in a direct lamella titled with the digit 4, and in a tranfverfe one with 7. The box fitted to receive thefe lamellæ is of a cubical form; fomething more than eleven inches wide and nearly eight inches high; fee Fig. VII. Two of its contiguous fides, which we fhall diftinguifh by the names of left and right, are divided

divided into twenty parts, each equal in length to the breadth of ten lamellæ, and in height to the thickneſs of a direct and of a tranſverſe lamella alternately. The greater diviſions on the left ſide are cut out, and the leſs on the right ſide. Into the box through each of the former, with their titled ends foremoſt, ten direct lamellæ of the ſame title are inſerted with their untitled ends foremoſt, and an equal number of the tranſverſe ones of the ſame title, through each of the latter. Thoſe titled *o* are in the uppermoſt diviſions, and thoſe titled 1, 2, 3, *&c.* in the reſpective diviſions below.

Multiplication by the promptuary is performed as follows. THE firſt, or right hand, ſecond, third, &c. figure of the multiplicand is exhibited by the title of a lamella taken from the firſt, or right hand, ſecond, third, &c. column of the left ſide of the box and placed on its lid exactly above and as it lay in that column. The empty ſpace, if any, towards the left is to be covered with blank lamellæ. The firſt, or right hand, ſecond, third, &c. figure of the multiplicator is exhibited by the title of a lamella taken from the firſt, or left hand, ſecond, third, &c. column of the right ſide of the box and placed on the former lamella exactly above, and as it lay in that column. The remaining ſpaces, if any, towards the right are to be covered with blank lamellæ. All the uſeful multiples on the direct lamellæ appear through the *feneſtellæ*, and all the uſeleſs multiples are hid. All the numbers beginning at the corner next the firſt or right hand figures of the multiplicand and multiplicator, lying between the united ſtrong diagonals, are to be added ſeverally; the right hand figures of the ſums, marked down; and 1 for every 10, carried to the next place, till we come to the oppo-

ſite

ſite corner: and the work is done. This operation, we truſt, is deſcribed with ſufficient accuracy and plainneſs to ſuperſede the neceſſity of an example. In order that diviſion may be performed by the Promptuary, it muſt firſt be converted into multiplication by means of tables dreſſed on purpoſe, or of tables of the ſines, tangents and ſecants, conſtructed on the hypotheſis of the radius being equal to unity, followed by a certain number of Zeros. That this may be accompliſhed by theſe laſt, look for the co-ſecant, or co-tangent of the arc which has the diviſor for its ſine or tangent. Make the co-ſecant or co-tangent found the multiplicand, and the dividend the multiplicator; or converſely. Find the product by the promptuary as above directed. This product, a number of the right hand figures according to the number of zeros in the ſquare of the radius being marked off as decimals, is the quotient required. The reaſon of this is obvious: the co-ſecants or co-tangents being third proportionals to the ſines or tangents and the radius or unity; to multiply any number by one of the two firſt, or to divide it by the correſponding one of the two ſecond of theſe lines, is one and the ſame thing.

LOCAL ARITHMETIC.

LOCAL Arithmetic, another ingenious invention of Napier, is the art of calculating by means of counters properly placed on a chess board, or similar table. Two contiguous margins (which we shall distinguish by the names of left or inferior, and right or lateral) of that table, are divided into a number of parts equal to that of their adjoining squares. The inferior divisions beginning at the right and the lateral at the left have successively inscribed in them the successive terms of the geometrical progression 1, 2, 4, 8, 16, 32, &c. which are called local numbers.

COMMON numbers are reduced to local by subtraction, and local numbers to common by addition. The common number 1875, for example, expressed in local numbers will be found to be 1024; 512; 256; 64; 16; 2 and 1: and *vice versa*. The first of these reductions is necessary before, and the second after any of the operations of common arithmetic are performed by this contrivance. By the help of a very simple table, reduction may be performed with ease and expedition.

To Add. PUT a counter for each local number in the proper place on the lateral or on the inferior margin of your table. For every two counters found in the same place, put one in the next higher, after removing *them*. Repeat this till no place shall contain more than one counter. The counters left indicate the numbers required. Thus let it be required to find the sum 1875; 258, and 1099. I put the coun-

ters

ters at 1024; 512; 256; 64; 16; 2 and 1, the local numbers of the firſt; at 256, and 2, thoſe of the ſecond; and at 1024; 64, 8, 2, and 1, thoſe of the third. At 1.—I find two counters which I remove, and put a counter at 2 where I find other three. I take up theſe four and put two, in the next place 4 &c. and proceeding in this manner I find at laſt a counter at each of the following numbers 2048; 1024; 128, and 32, which form 3232 the ſum ſought.

To Subtract. PUT a counter for each local number of the greater of the two quantities, at its proper place, a little to one ſide, on the inferior margin; and one for each of the local numbers of the leſs of the two quantities, at its proper place, a little to the other ſide, on the ſame margin. Remove the counters found on oppoſite ſides of the ſame place. Change the ſide of the loweſt counter remaining; take off that above it; and put a counter in each place between them. Remove as before. Repeat this till there ſhall be no counters on one of the ſides of the margin; and thoſe on the other will indicate the remainder. Let it be propoſed, for example, to ſubtract 1099 from 1875. I put counters at 1024; 64; 8; 2 and 1, to the left of the lateral margin, and at 1024; 512; 256; 64; 16; 2 and 1, to the ſlit of that margin. Finding a counter on each ſide of the numbers 1024; 64; 2, and 1, I remove them. My loweſt counter is to the left of 8. I put it to the right and take up 16. above it; as there are no intermediate places, and as the remaining counters are on the ſame ſide of the margin my operation is finiſhed. The remainder is 512; 256, and 8; or 776.

MULTIPLICATION,

MULTIPLICATION, Divifion, and the extraction of the fquare root, may alfo be performed on the margin: but they are performed with much greater eafe and clearnefs on two contiguous margins and the fquares of the table. On thefe laft the counters have two different movements; the one parallel to the fides like that of the tour, and the other diagonal like that of the bifhop, on the chefs board. Every fquare of the table is faid to have for its value one of the equal numbers (on the two margins) between which it lies diagonally. The two fides of a fquare formed by counters in the area of the table, parallel to the inferior and lateral margins, we fhall call a Gnomon: this gnomon confifting of 3, 5, 7, &c. counters is faid to be congruous when its value can be fubtracted from the numbers left marked upon the margin.

To Multiply. MARK with counters the local multiplicator in the inferior and the local multiplicator in the lateral margin. From the middle of the marked places let points be fuppofed to move perpendicularly into the table, and put a counter at each interfection. Remove the counters on the margins. Bring the counters in the fquares of the tables to their values in one of the margins; add, if neceffary, and the work is done. Suppofe, for example, 19 is to be multiplied by 13. I mark with counters Fig. VIII. the numbers 1, 2, and 16, on the inferior and the numbers 1, 4, and 8, on the lateral margin, having placed other counters rectangularly in the table, I remove the marginal ones. Thofe other counters I bring up, one by one, to their proper places in the lateral margin; and, after adding, I find a counter at each of of the following numbers, 128; 64; 32; 16; 4; 2, and 1, which form my product, 247.

To Divide. MARK with counters the local dividend in the lateral, and the local divisor in the inferior margin, beginning at the square where a point, descending diagonally from the angle above the highest number of the dividend, would intersect a point ascending perpendicularly from the highest number of the divisor; place a series of counters parallel to the divisor: If this series is equal or inferior in value to the higher number of the dividend subtract it from them; and if otherwise, bring it down one, two, &c. steps and subtract. Repeat the operation till either nothing, or at least a numberless than the divisor, shall remain on the lateral margin. These serieses point to the numbers that form the quotient. For example let it be required to divide 250 by 13. I mark, Fig. IX. the numbers 128; 64; 32; 16; 8; and 2, in the lateral, and 8; 4, and 1 in the inferior margin.

MY first series points to 16. I subtract it from the dividend and find remaining 32; 8, and 2.

MY next series pointing to 4 is too great to be subtracted, for which reason I bring it a step farther down.

AFTER subtracting, there remains 16. In the same manner my third series pointing to 2 I must bring to point to 1; which subtracted, there remains counters on the dividend at 2 and 1. My quotient is therefore 16; 2, and 1, or 19; and 2 and 1, or 3 over.

To extract the square root. Mark the number locally in the lateral margin. From the angle formed by the meeting of the inferior line with the lateral, let a point ascend diagonally till it arrive in a square of the same value with the highest number that can be subtracted from the number whose square root is sought. In this square place a counter, and subtract its value from the number marked in the margin. Form the congruous gnomons, which from the foresaid square have each their upper counter in a line perpendicular and their left hand inferior one in a line parallel to the lateral margin: and subtract their value one by one from the marked remainders. The counters, lying perpendicularly to either of the margins, point out the square root. Let it be proposed, for example, to find the square root of 2209; I mark the numbers 2048; 128, 32, and 1 on the lateral margin. Fig. X. Subtracting the value 1024 of the first counter placed in the table as directed, the remainders are 1024, 128, 32 and 1. From these taking the value 512 and 64 of the first congruous gnomon, there remain 512, 64, 32 and 1. From these taking the value of the second congruous gnomon 256, 64 and 16, there remain 64, 16, 8, 4 and 1: and from these taking the value of the fourth congruous gnomon, 64, 16, 8, 4 and 1, there remains nothing. The square root, as indicated by the direction of the counters in the table, is 32, 8, 4, 2 and 1, or 47.

What is above said will, it is hoped, be sufficient to give a clear idea of the form and use of those of Napier's arithmetical instruments, which seemed to him worthy of being communicated to the public. The reasons on which the different operations are founded, depending upon the construction of the machines and the obvious properties of numbers,

bers, muſt occur to every reader endowed with a moderate ſhare of attention. The hint of the rods, or *virgulæ numeratrices* and of the promptuary which is only an improvement of the rods, ſeems to have been taken from the *Abacus Pythagoricus* or common multiplication table. Napier's acquaintance with cheſs, the moſt ingenious of all games, and at which mathematicians are commonly the beſt players, occaſioned his diſcovery of the *Arithmetica localis.* The Promptuary, at leaſt for multiplication, is greatly preferable to the rods and the cheſs board; for the partial products of two numbers, conſiſting of even ten Figures each, may, by a little practice, be exhibited on that machine in the ſpace of a minute, and no numbers require to be written out, excepting the total product. Had the logarithms remained undiſcovered, the promptuary, in all probability, would have become univerſally familiar to thoſe who were engaged in tedious calculations. But to thoſe who are acquainted with the logarithms, Napier's arithmetical machines and thoſe afterwards invented, a few of which we ſhall enumerate, although the monuments of genius, muſt, in general, be regarded as mathematical curioſities of no uſe.

PERHAPS put into the hands of young people learning arithmetic, they might make them fond of that ſtudy.

SHICKARTUS in a letter to Kepler, written in the year 1623, informs him that he had lately conſtructed a machine conſiiſting of eleven entire and ſix mutilated little wheels, by which he performed the four arithmetical operations *, Paſcal, in the year 1642, at the age of nineteen,

* Kepl. Epiſt. p. 683.

teen, invented a machine with which all kinds of computations may be made without the pen, without counters, and without the knowledge of any rule of arithmetic. I have not been able to meet with any defcription of it. It muft however have been of a very complicated nature as its author was two years in bringing it to perfection, owing to the difficulty he found to make the workmen underftand him thoroughly *. The French writers agree in calling it admirable; † but the name of Pafcal perhaps does it more honour than it deferves. This machine is preferved in the cabinet of the king of France and in thofe of a few others ‡.

The Marquis of Worcefter, a man of genius but a plagiary, mentions in his fcantlings of inventions, publifhed in the year 1655, an inftrument whereby perfons ignorant in arithmetic may perfectly obferve numerations and fubtractions of all fums and fractions §. Whether he here refers to fome of Napier's inftruments, to Gunter's fcale, of which I fhall afterwards fpeak, or to fome invention of his own is uncertain.

About the year 1670, ** Sir Samuel Moreland contrived two arithmetical inftruments; one for addition and fubtraction, and the other for multiplicaton, divifion, and the extraction of the fquare, cube, and fquare cube roots, the defcription of which he publifhed at London, anno 1671 ††.

Much

* Les hommes illuftres par Perrault vie de Pafcal. † Bayle Chauffepie, Baillet, Perrault, &c.

‡ Pref. Penfees de Pafcal. § N° 84. Glag. Edit. 1767.

** Moreland's Inftrument of excellent ufe as well at fea as at land, invented and varioufly experimented in the year 1670, Lond. 1671. Fol.

†† See alfo Phil. Tranfact. N° 94. p. 6048.

Much about the ſame time, Mr George Brown, afterwards Miniſter of Kilmaures in Scotland, invented a machine which, in his account of it publiſhed at Edinburgh in the year 1700, he calls the *Rotula Arithmetica*. This machine conſiſts of a box containing a circular plane moveable on a center pin and fixed ring, whoſe circles are deſcribed from the ſame center. The outermoſt circular band of the moveable, and the innermoſt of the fixed, are each divided into a hundred equal parts, and theſe parts are numbered 0, 1, 2, 3, &c. Upon the ring there is a ſmall circle having its circumference divided into ten equal parts, furniſhed with a needle which ſhifts one part at every revolution of the moveable. By this ſimple inſtrument are performed the four arithmetical operations not only of integers but even of decimals finite and infinite *.

Some time before Mr Brown's little book appeared, Mr Glover had publiſhed a *Roue Arithmetique* ſimilar to the Rotula but not ſo perfect. It would appear however that that gentleman had got ſome hints of the conſtruction of the Rotula from a brother of his own who had been one of Brown's pupils in 1674 †.

In the year 1725, an inſtrument invented by M. de l'Epine of a more ſimple conſtruction and eaſier in its operations than Paſcal's; in 1730, another by Mr Boiſſendeau, by which calculation is performed without writing; and in 1738 a third by Mr Rauſlin, conſiſting of rods different from thoſe of Napier, were approved of by the French academy ‡.

* One of theſe machines is in the library belonging to the faculty of advocates at Edinburgh.

† Rot. Arithm. Pref. ‡ See the Paris memoirs for theſe years. Hiſtoire.

Sam Reyer invented, at what time I have not been able to learn, a kind of ſexagenal rods in imitation of Napier's, by which ſexagenary arithmetic is eaſily performed *.

I have an arithmetical machine which came into my poſſeſſion from my uncle George Lewis Erſkine who, though born deaf, by the aſſiſtance of the learned Henry Baker of the Royal Society at London, acquired not only the uſe of ſpeech and the learned languages but a deep acquaintance with uſeful literature. This machine conſiſts of a ſmall ſquare box furniſhed with ſix cylinders moveable round their axes. Upon each of theſe cylinders, which are only Napier's rods, are engraven the ten digits, and their multiples. From a perpetual almanac on the out ſide of the box, it would appear that this machine was conſtructed in the year 1679.

SECTION

* See Chamber's Diction. Article Arithmetic.