The doctrine of projectiles demonstrated and apply'd to all the most useful problems in practical gunnery

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Part II.

PART II.

The DOCTRINE *of* PROJECTILES *apply'd to* PROBLEMS *of* PRACTICAL GUNNERY.

HAVING in the foregoing *Sections* explain'd and demonſtrated the *Theory* of *Projectiles*, and the *Laws* of *Motion* and *Gravitation*, as far as was requiſite to the Knowledge of it; I now proceed to apply that *Theory* to the *Art* of *Gunnery* in all the moſt uſeful Examples that I think an *Ingeneer* can have Occaſion for, with reſpect to the projectile Part, which is all that is deſign'd here; but firſt, it will be convenient to ſhew how to direct a Piece of Ordnance according to any Degree of Elevation or Depreſſion.

Let

Let *AB*, (*Fig.* 18.) be a Ruler of about five Foot long, and let each Quadrant of the Semicircle *ADC*, *qDC* be divided into 90°, and mark'd at each ten, thus, 10, 20, 30, *&c.* counting from *D* towards *A*, and *q*, the Spring *Bfg* is made of Steel and fixt to the End *B*, in order to keep the Ruler close to the lower Side of the Cylinder, or Bore of the Piece, that so it may be parallel to the Axis of the Cylinder when it is thrust into it: Now it is plain, by the *Fig.* that if the Piece is elevated any Number of Degrees from the horizontal Line *HO*, the Thread of the Plummet *CK*, will cut the Number of Degrees on the Limb of the Quadrant, for the Angle *CBO*=*KCD*, because *BCK* is the Complement of each to a Quadrant. And thus may a Mortar, or any other Piece of Ordnance, be elevated or depress'd to any Degree, which is call'd laying them to pass.

CHAP. I.

Of PROJECTIONS *made on the* PLANE *of the* HORIZON.

CASE I.

A Piece whoſe greateſt Randon is *AM* (*Fig.* 13.) = 8000, Quere her Elevation to ſtrike an Object at *m*, whoſe Diſtance from the Piece is *Am* = 5280; as alſo the Height *CV*, and Sublimity *VK*, of the Projection.

$\frac{1}{2}AM = Af = 4000$, $\frac{1}{2}Am = AC = 2640$, then, $Af : AC :: R : S$, $AfC = 41°. 18'$, and $90° - \frac{1}{2}AfC = 69°. 21'$, the higher Elevation, alſo $\frac{1}{2}AfC = 20°.$

39′ the lower Elevation, and, $R:S,CAf::Af:fC=3005$, $\frac{Af+fC}{2}=CV=3502.5$ the Height, also $Af-VC=VK=497.5$ the Sublimity; all which is obvious from the *Figure* and *Scholium* 3d.

CASE II.

A Piece whose greatest Randon is AM (*Fig.* 13.) $=8000$, Quere her Randon at 69°. 21′, $=CAt$, as also, the Height and Sublimity of the Projection.

$R:S,2|\overline{CAt}::AM:Am=5280$. the Distance required; (*Cor.* 6. *Theo.* 10.) the rest are found, as *per Case* 1.

CASE III.

The greatest Randon of a Piece being AM(*Fig.* 13.)$=8000$, and Height of of a Projection made at Random $CV =$ 3502.5 given, to find the Sublimity, Direction, and Amplitude.

$\frac{1}{2} AM = Af = CK$, and $CK - CV = VK = 497.5$, the Sublimity, and $CV - VK = fC = 3005$; (*Cor.* 4. *Theorem* 7.) then $Af : fC :: R : S$, $CAf = 48°.42'$, and $\frac{90 + CAf}{2} = 69°.21' = CAt$, the Direction sought; also $R : S, 2\overline{CAt} :: AM : Am = 5280$, the Amplitude.

CASE IV.

The greatest Randon AM (*Fig.* 13.) $= 8000$, and Sublimity of a Projection $VK = 497.5$ given, to find the Height CV, Elevation CAt, and Amplitude AM.

$\frac{1}{2} AM = Af$, and $Af - VK = CV =$ 3502.5 the Height, and (by *Cor.* 4. *Theo.* 7.) $CV - VK = fC = 3005$, and having Af and fC, the rest are found, as *per Case* 3.

CASE V.

The Amplitude Am (*Fig.* 13.) = 5280, and Elevation CAt equal 69°. 21′ given, to find the Impetus Af, Height CV, and Sublimity VK.

$S, 2\,\overline{CAt} : R :: Am : AM =$ gr.Ran. = 8000, and ½ gr. Ran. = $Af = 4000$ the Impetus; (*Cor.* 6 & 7. *Theo.* 10.) then, $2CAt - 90° = CAf$. *per Fig.* and $R : S, CAf :: Af : fC = 3005$; also (*per Schol.* 3.) $\frac{Af + fC}{2} = CV = 3502.5$ the Height, and $Af - CV = VK = 497.5$ the Sublimity of the Projection.

CASE VI.

The Amplitude Am (*Fig.* 13.) = 5280, and Height $CV = 3502.5$ given,

ven, to find the Elevation *CAt*, Impetus *Af*, and Sublimity *VK*.

$\frac{1}{2}$ *Am* = 2640 = *AC*, and 2 *CV* = *Ct*; (*Cor.* 2. *Theo.* 9.) then *AC* : *Ct* :: *R* : *T*, *CAt* = 69°. 21′ the Direction sought, and having the Amplitude *Am*, and Direction *CAt*, the rest are found, as *per Case* 5.

CASE VII.

The Amplitude *Am* (*Fig.* 13.) = 5280, and Sublimity *VK* = *Cu* = 497.5 the Height of the lower Projection (*Cor.* 9. *Theo.* 10.) being given, to find the Direction *CAt*, Impetus *Af*, and Height *CV*.

$\frac{1}{2}$ *Am* = *AC*, and 2 *Cu* = *Ci*, (*Cor.* 2. *Theo.* 9.) then *AC* : *Ci* :: *R* : *T*, *CAi* = 20°. 39′, and 90° − *CAi* = 69°. 21′ = *CAt* the Direction sought, (*Cor.* 8. *Theo.* 10.) and having the Direction and

and Amplitude, the rest are found as *per Case* 5.

CASE VIII.

The Elevation $CAt = 69°. 21'$ (*Fig.* 13.) and Height $CV = 3502.5$ being given, to find the Amplitude Am, and Sublimity VK.

$2CV = Ct$, (*Cor.* 2. *Theo.* 9.) and $S, CAt : S, CtA :: Ct : CA = 2640$, and $2CA = Am = 5280$ the Amplitude, the rest are found as *per Case* 5.

CASE IX.

The Elevation CAt (*Fig.* 13.) $= 69°. 21'$, and Sublimity $VK = 497.5 = Cu$ the Height of the lower Projection (*Cor.* 9. *Theo.* 10.) being given, to find Am, Af and CV, the Amplitude, Impetus, and Height.

90°

$90° - CAt = 20°. 39' = CAi$, the lower Elevation, (*Cor.* 8. *Theo.* 10.) and $2Cu = Ci$. (*Cor.* 2. *Theo.* 9.) then $S, CAi : S, CiA :: Ci : CA = 2640$, and $2AC = Am = 5280$, the Amplitude ſought, alſo $S, 2\overline{|AiC} : AC :: R : Af = 4000$ the Impetus, and $Af - VK = CV = 3502.5$ the Height of the Projection.

CASE X.

The Height $CV = 3502.5$, and Sublimity $VK = 497.5$ given, to find the Impetus Af, Elevation CAt, and Amplitude Am. (*Fig.* 13.)

$CV + VK = CK = Af$, the Impetus (*Cor.* 3. *Theo.* 10.) $CV - VK = 3005 = fC$, (*Cor.* 4. *Theo.* 7.) then $Af : fC :: R : S, fAC = 48°. 42'$, and $\frac{90° + fAC}{2} = 69°. 21' = CAt$ the Elevation, alſo $R : S, 2\overline{|CAt} :: 2Af : Am = 5280$ the Amplitude ſought.

Note,

Note, If nothing is required of the *Ingeneer* but to reach the Object, let 45° be the constant Elevation, and the absolute Forces which are as the square Roots of the Impetuses (*Cor.* 4. *Theo.* 5.) are as the square Roots of the Distances; (*Cor.* 7. *Theo.* 10.) but the absolute Forces are as their adequate Causes, the Gun-Powder, whence the Charges requisite to carry equal Shot different Distances, are as the square Roots of those Distances.

CASE XI.

The greatest Randon of a Cannon-Royal being 8000 Paces, and her Requisite of Powder for that Randon 28 Pounds, Quere the Requisite of Powder to carry her Shot 3600 Paces.

$\sqrt{8000} : \sqrt{3600} :: \overset{lb}{28} : \overset{lb}{18\frac{2}{3}}$, the Charge required. *Note*, If all the Powder exerted its Force in the Chace, or Bore, of the Piece, this Proportion wou'd wou'd be Mathematically true; but ſome of it, eſpecially in high Charges, does not take Fire till out of the Mouth of the Piece, which will occaſion ſome Variation, &c. By Randons made at this Elevation much Gun-Powder may be ſav'd, and the Ingeneer more certain of hitting the Object than at any other; for an Error of a Degree above, or below 45°, will not produce a ſenſible one in the Diſtance, becauſe the Sines within a Degree of 90°, differ very little from the Radius, and thoſe Sines near the Elevation of 45°, are as the Diſtances.

CASE XII.

The Impetus of a Cannon Royal being 12000 Feet, Quere the Celerity of her Shot at parting from the Mouth of the Piece, or at meeting with the Object on the Plane of the Horizon, for in both Caſes, the Celerity is equal to what wou'd be acquir'd by falling through the given Impetus. (*Theo.* 10.)

$8 \times \sqrt{12000} = 876$ the Celerity required, or the Feet the Ball moves through in a Second of Time: this follows from *Theo.* 5. and its *Corolaries*, and becauſe a heavy Body falls through 16 Feet in a Second of Time. For, from theſe $\sqrt{16} : 32 :: \sqrt{12000} : 876$ the Celerity, and becauſe 4, and 32 are the two firſt Terms in the Ratio, the ſquare Root of any Impetus multiply'd by 8, will give the Celerity.

CASE XIII.

The Impetus of a Piece being 12000 Feet, and the Weight of her Shot 63 Pounds, Quere the abſolute Force, or Momentum of the Ball in known Weight.

By the laſt Caſe, $8 \times \sqrt{12000} = 879$ $= C$ the Celerity, and (*per Scholium* 2. to the Laws of Motion) $B \times C \times 1.3 =$ M, *M*, that is, in Numbers, $63 \times 876 \times 1.3 = 71744 =$ the Momentum or absolute Force in Pounds, which is about $32\frac{1}{2}$ Tuns *Averdupois*. *Note*, because 8, and 1.3 are constant Multipliers, $8 \times 1.3 = 10.4$ will be a constant Multiplier.

CASE XIV.

A Piece whose greatest Randon is 8000, and whose absolute Force is 71744 Pounds, Quere with what Force she will strike an Object whose Distance is 5600, and whose Surface reclines, or leans directly from the Perpendicular 42°.

8000 : 5600 :: *R* : *S*, 44°. 26′ double the Elevation, consequently the Ball strikes the Plane of the Horizon at an Angle of 22°. 13′, and the Surface of the Object, which reclines from the Perpendicular 42°, at an Angle of 70°. 13′; then (*per Theo.* 2.) *R* : *S*, 70°.

$13'$ :: 71744 : 67512 the Forcc required.

CASE XV.

The greateſt Randon of a Piece being 8000, Quere at what Diſtance ſhe will ſtrike an Object with the greateſt Force poſſible whoſe Surface reclines 42°.

Since the Surface reclines 42°, it's plain, the Direction to ſtrike it at Right Angles, which is the Direction of the greateſt Force, is 42°; then *R* : *S*, 2 |42° :: 8000 : 7956, the Diſtance required. Hence it's eaſy to obſerve, that if the Surface of an Object recline 45°, the greateſt Force it can be ſtruck with is from the greateſt Diſtance, and the contrary.

Note, If the Ingeneer has Field-Room, and can vary his Poſitions and Diſtances as he pleaſes, he may ſtrike an an Octject, whoſe Surface is in a given Poſition, at any Angle, and conſequently with any Force he pleaſes, not exceeding the greateſt Force of his Piece.

CASE XVI.

The greateſt Randon of a Piece being 8000, Quere at what Diſtance ſhe will ſtrike an Object, whoſe Surface reclines 42°, with a Force which ſhall be to the greateſt Force of the Piece as 3 to 4.

4 : 3 :: *R* : *S*, 48°. 35′ = the Angle the Object muſt be ſtruck at, (*Theo.* 2.) and ſince its Surface reclines 42° from the Perpendicular, its Complement 48°, taken from 48°. 35′ leaves 35′ for the lower Elevation, and 48°. 35′ − 42° = 6°. 35′ the Complement of 83°. 25′, the higher Elevation. Then *R* : *S*, 1°. 10′ :: 8000 : 163, the Diſtance at the

the lower Elevation, and $R : S, 2\,|\overline{80^\circ.25'} :: 8000 : 1822$ the Diſtance at the higher Elevation.

CASE XVII.

A Piece whoſe gr. Ran. with 28 Pounds of Powder is 8000, Quere her Charge to ſtrike an Object at Right-Angles whoſe Diſtance is 1822, and whoſe Surface reclines 42°.

It's plain, the Direction which ſtrikes this Object at Right-Angles muſt be an Elevation of 42°. Then $S, 2\,|\overline{42^\circ} : R :: 1822 : 1832 =$ the greateſt Randon for the Charge required, and $\sqrt{8000} : \sqrt{1832} :: \overset{lb}{28} : \overset{lb}{13.3}$ the Charge requir'd.

CHAP.

CHAP. II.

Of Projections *made on* Aſcending Planes, *and to* Perpendicular Heights.

CASE XVIII.

THE greateſt Randon of a Piece on the Plane of the Horizon being 8000, Quere her greateſt Randon on a Plane whoſe Aſcent is 6°. 30′.

$R + S, 6°. 30' : R :: 8000 : 7186.4$ the Randon required. (*per Cor.* 4. *Theo.* 12.) *Note*, The moſt exact and expeditious Way to perform this and the like Operations is by the natural Sines; for

for $R + S$, 6°. 30′ = 11132032, and 8000 × R = 80000000000, then 11132032)80000000000(7186.4; but if any one choose to do it by *Logarithms*, he must strike off 4 Figures of the first Term for Decimals, and take 3 for the Log. of the Radius, then will all the Terms be had in the common Tables.

CASE XIX.

The greatest Randon on a Plane whose Ascent is 6°. 30′ being 7186.4, Quere the greatest Randon on the Plane of the Horizon.

$R : R + S$, 6°. 30′ :: 7186.4 : 8000 the Randon required.

CASE XX.

The greatest Randon on a Plane whose Ascent is 6°. 30′ being 7186.4; Quere

Quere her greateſt Randon on a Plane whoſe aſcent is $10^{\circ}. 20'$.

By *Cor.* 5. *Theo.* 12. $R+S$, $10^{\circ}. 20'$: $R+S$, $6^{\circ}. 30'$ :: 7186.4 : 6783, the Randon required.

CASE XXI.

The greateſt Randon of a Piece on the Plane of the Horizon being 8000, Quere the greateſt Height ſhe can reach on the Perpendicular Y *K* (*Fig.* 11.) whoſe horizontal Diſtance is *A*Y = 7143.

By *Cor.* 6. *Theo.* 12. $\frac{1}{2}$ gr. Randon $- \frac{AY^q}{2 \text{ gr.Ran.}} = XY = 811$ the Height ſought. Or by the *Logarithms* thus, from the double Log. of *A*Y, take the Logarithm of double the gr. Randon the Number anſwering the Remainder taken from $\frac{1}{2}$ the gr. Ran. is the Height required.

CASE XXII.

The greatest Randon on the Plane of the Horizon being 8000, Quere the Direction to strike the greatest Height possible on the Perpendicular Y*X*, (*Fig.* 11.) whose horizontal Distance is *A*Y = 7143.

Find Y*X*, that greatest Height by the last *Case*, then *X*Y : *A*Y : : *R* : *T*, *AX*Y = *T*, 83°. 31′ = *T*, *XAZ*, and 90°. − ½ *XAZ* = 48°. 15′, the Direction required.

CASE XXIII.

A Piece whose horizontal Randon at the Elevation Y*AI* = 64°. 02′ (*Fig.* 19.) is *Am* = 6060, Quere her Randon at the same Elevation on the Plane *A*X, whose Ascent is Y*AX* = 6°. 30′.

T, Y*AI* − *T*, Y*AX* = *T*, *AmX* = *T*, 62°. 44′, (*Cor.* 1. *Theo.* 13.) whence

Y*AX*

Y $AX + AmX = 69°. 14'$, which gives $AXm = 110°. 46'$, but the Sine of the Sum of any two acute Angles is always equal to the Sine of the obtuſe Angle, therefore $S, 69°. 14' : S, 62°. 44' :: Am : AX = 5761$ the Diſtance required.

CASE XXIV.

A Piece whoſe Randon at $64°. 02' =$ Y AI, is $AX = 5761$, on a Plane whoſe Aſcent is Y $AX = 6°. 30'$, Quere her Ran. on the Plane of the Horizon at the ſame Elevation.

T, Y$AI - T$, Y$AX = T, AmX = T, 62°. 44'$, then $S, AmX : S, AXm :: AX : Am = 6060$.

CASE XXV.

A Piece whoſe Randon at 40°. on the Plane of the Horizon is AM (*Fig.* 19.)

19.) = 7580, Quere her Ran. at 64°. 02′ = Y *A I*, on a Plane whose Ascent is Y *A X* = 6°. 30′.

S, 2 $\overline{|40°}$: *S*, 2 $\overline{|64°.02'}$: : *AM* : *Am* = 6060, then, *T*, Y *AI* − *T*, Y *AX* = *T*, *AmX* = *T*, 62°. 44′, lastly *S*, *AXm*; *S*, *Am X* : : *Am* : *AX* = 5761.

CASE XXVI.

A Piece whose Randon at 64°. 02′ = *MAI* (*Fig.* 23.) is *AX* = 5761, on a Plane whose Ascent is *M A X* = 6°. 30′, Quere her Randon on the same Plane at 21°. 09′ = *M Ai*.

T, *M Ai* − *T*, *M A X* = *T*, *AMX*, whence we have *AXM*, then *S*, *AMX* : *S*, *A XM* : : *A X* : *AM* = 6060, and *S*, 2 $\overline{|MAI}$: *S*, 2 $\overline{|MAi}$: : *A M* : *Am* = 5180, also *T*, *M Ai* − *T*, *M A x* = *T*, *A mx* = *T*, 15°. 16′; lastly, *S*, *Ax m* : *S*, *Amx* : : *Am* : *Ax* = 3678, the Randon required.

CASE

CASE XXVII.

A Piece whoſe Randon at $mAi=21^\circ.\,09'$, on a Plane elevated mAx, or $MAX=6^\circ.\,30'$, is $AX=3678$, Quere her Randon at $MAI=64^\circ.\,02'$, on a Plane whoſe Aſcent is $MAD=13^\circ$.

$T,mAi-T,mAx=T,Amx=T,15^\circ.\,16'$, whence we have Axm, then $S,Amx : S,Axm :: Ax : Am=5180$. and $S,2\,|\overline{mAi} : S,2\,|\overline{MAI} :: Am : AM=6060$; alſo $T,MAI-T,MAD=T,AMD$, whence we have ADM, as before, and $S,ADM : S,AMD :: AM : AD=5533$ the Randon required.

CASE XXVIII.

A Piece whoſe Randon at $MAI=64^\circ.\,02'$ (*Fig.* 19.) on the Plane of the Horizon is $AM=6859$, Quere how high ſhe will ſtrike an Object with the ſame ſameDirection on a Perpendicular Y *K*, whoſe horizontal Diſtance is *A* Y = 6478.

T, *MAI* : *R* : : *AM* : *L* = 3340 = the Latus Rectum ; (*Lem.* 2.) then *AM* − *A*Y = 381 = Y*M*, and *L* : *A*Y : : Y*M* : YX = 739 (*Lem.* 1.) the Height required.

CASE XXIX.

A Piece at *A* (*Fig.* 19.) elevated *MAI* = 64°. 02′, ſtrikes an Object at X, whoſe perpendicular Height above the Plane of the Horizon is YX = 739, and whoſe horizontal Diſtance is *A*Y = 6478, Quere her Randon on the Plane of the Horizon with the ſame Elevation.

*A*Y : YX : : *R* : *T*, Y*A*X = *T*, 6°. 30′, and *T*, *MAI* − *T*, Y*A*X = *T*, *AM*X, = *T*, 62°. 44′, whence YX*M* = 27°. 16′; then *S*, *AM*X : *S*, YX*M* : : YX : Y*M* = 381.

$= 381$. and $AY + YM = AM = 6859$ the Randon required.

CASE XXX.

A Piece whose horizontal Amplitude at 64°. 02′ is AM (*Fig.* 7.) $= 6859$, strikes an Object with the same Direction on the Top of the Perpendicular $YG = 739$. Quere the horizontal Distance AY, or AO of that Perpendicular.

$R : T, 64°.02' :: AC (= \frac{1}{2}AM) : 2CV$, (*Cor.* 2. *Theo.* 9.) whence $CV = 3521$, and $CV - YG = VE = 2782$, and (*per Theo.* 6.) $CV : CM^q :: EV : EG^q$, whence $EG = CY = 3048.5$, and $AC + CY = 6478$, the greatest Distance, and $AC - CY = AO = 381$, the least Distance.

CASE XXXI.

A Piece at A (*Fig.* 19.) elevated in $AI = 64°.02'$ strikes an Object at X whose

whoſe Perpendicular Height above the Plane of the Horizon is Y X = 739, and whoſe horizontal Diſtance is *A*Y = 6478, Quere her Direction to ſtrike an Object on the Plane of the Horizon whoſe Diſtance is *AM* = 7812.

*A*Y : Y X : : *R* : *T*, Y *A*X = 6°. 30′, and *T*, *mAI* − *T*, Y *A* X = *T*, Y *m* X = *T*, 62°. 44′, whence we have Y X*m* = 27°. 16′, then *S*, Y *m* X : *S*, Y X*m* : : Y X : Y *m* = 381, and *A*Y + Y *m* = *Am*, laſtly *Am* : *AM* : : *S*, 2 |$\overline{mAI}$: *S*, 63°. 42′, ½ of which 31°. 51′ = the Direction required.

CASE XXXII.

A Piece whoſe gr. Ran. on the Plane of the Horizon is 8000, Quere her Direction and greateſt horizontal Diſtance *A*Y (*Fig.* 11.) from which that Piece can ſtrike an Object on the Top of the Perpendicular Y X = 2179.

gr.

gr. Ran: − Y X = 5821 = *A* X, (*Cor.* 3. *Theo.* 12.) and *A X* : X Y : : *R* : *S*, Y *A* X = 21°. 58′, whence *A X* Y = 68°. 02′, and 90°. − ½ *A* X Y = 55°. 59′ = Y *A I*, the Direction, (*Cor.* 2. *Theo.* 12.) then *R* : *S*, Y X *A* : : *A* X : *A* Y = 5398 the Diſtance ſought.

CASE XXXIII.

The Perpendicular Height of an Object X Y = 2179, and horizontal Diſtance *A* Y = 5398 being given, to find the leaſt Impetus that poſſibly can reach that Object.

$\sqrt{AY^q + XY^q} = 5821 = AX$, (47. *El.* 1.) and $\frac{AX + XY}{2} = 4000 = AZ$ the Impetus required. (*Cor.* 3. *Theo.* 12.)

CASE XXXIV.

A Piece whoſe Impetus is *Af*, (*Fig.* 19.) = 4000, Quere her Direction to

P ſtrike

ſtrike an Object whoſe Diſtance is $AX = 5761$ on a Plane whoſe Aſcent is $YAX = 6^\circ.\ 30'$.

$R : S, YAX :: AX : YX = 652$, and (*Cor.* 11. *Theo.* 12.) $AX : 2Af - XY :: XY : Aq = 831$, then $\frac{AX + Aq}{2} = AP = 3296$, and $Af : AP :: R : S$, $AfP = 55^\circ.\ 30'$, alſo $AfP - YAX = Afd = fAZ$, and $90^\circ - \frac{1}{2} fAZ = 65^\circ.\ 30'$, $= YAl$ the higher, and $\frac{1}{2} fAZ = 24^\circ.\ 30'$ the lower Elevation. (*Ar.* 3. *Scholium* 3.)

CASE XXXV.

A Piece whoſe Randon at $32^\circ.\ 28'$, on a Plane whoſe Aſcent is YAX, (*Fig.* 19.) $= 6^\circ.\ 30'$, is $AX = 6520$, Quere her Direction to ſtrike an Object whoſe horizontal Diſtance is 6000.

$T, 32^\circ.\ 28' - T, 6^\circ.\ 30' = T, 27^\circ.\ 34' = T, AmX$, whence $AXm = 145^\circ.\ 56'$, 56′, then *S*, *AmX* : *S*, *AXm* :: *AX* : *Am* = 7892, the horizontal Amplitude at 32°. 28′; laſtly 7892 : 6000 :: *S*, 2|32°. 28′ : *S*, 43°. 32′, ½ of which = 21°. 46′ the Direction required.

CASE XXXVI.

A Piece whoſe Impetus is *Af* (*Fig.* 19.) = 4000, Quere her Direction to ſtrike an Object on the Top of a Perpendicular whoſe Height is Y*X* = 652, and whoſe Diſtance, in a ſtraight Line, is *A*X = 5761.

AX : *X*Y :: *R* : *S*, Y*AX* = 6°. 30′ = *dfP*, (*Cor.* 11. *Theo.* 12.) and *A*X : 2*Af* − Y*X* :: Y*X* : *Aq* = 831, alſo $\frac{Af+Aq}{2}$ = *AP*, then *Af* : *AP* :: *R* : *S*, *AfP*, and *AfP* − *dfP* = *Afd* = *fAZ*; laſtly, 90°. − ½*fAZ* = 74°. 41′ the Direction ſought. (*Ar.* 3. *Scholium* 3.)

CASE XXXVII.

A Piece whose horizontal Amplitude is *AM* (*Fig.* 19.) = 6859, strikes an Object with the same Direction on the Top of a Perpendicular whose Height is YX = 739, and whose Horizontal Distance is *A*Y = 6478, Quere that Direction.

Am − *A*Y = Y*m* = 381, then, (*Lem.* 1.) YX : Y*m* :: Y*A* : *L* = 3340, and (*Lem.* 2.) *L* : *Am* :: *R* : *T*, 64°. 02′, the Direction sought.

CASE XXXVIII.

A Piece whose horizontal Amplitude is *AM*, (*Fig.* 19.) = 7892, strikes an Object with the same Direction, whose Distance is *AX* = 6520, on a Plane whose Ascent is *MAX* = 6°. 30′, Quere that Direction.

Since

Since $6^\circ.30'$ is one Angle of the Triangle AXM, $86^\circ.45'$ = half the Sum of the other two, and $AM + AX : AM - AX :: T, 86^\circ.45' : T, 59^\circ.11'$ = half the Difference of the other two Angles, whence $86^\circ.45' - 59^\circ.11' = 27^\circ.34' = AMX$, and (*per Theo.* 13.)

$T, AMX + T, MAX = T, 32^\circ.28'$, the Direction sought.

CASE XXXIX.

The Impetus Af (*Fig.* 19.) = 13068 Feet, and the Height of the Object YX = 2217, together with the Weight of the Shot = 63 Pounds given, to find the absolute Force at X.

$Af - YX = KX = 10851$ = the Impetus at X, (*Theo.* 10.) and $10.4 \times 63 \times \sqrt{10851} = 68245$ the Force required in Pounds. (*Prop.* 13.)

CASE

CASE XL.

The Direction Y *At* (*Fig.* 21.) = 64°. 02′, and Angle Y *Ax* = 6°. 30′, together with the abſolute Force at the Point *x* = 68245 given, to find the Force wherewith the Plane *Ax*, or the Surface of any Object at *x*, of a given Inclination, may be ſtruck by that Direction and Impetus.

Make *dx* parallel to *AM*, and (*Theo.* 14.) *T*, Y *At* − 2 *T*, Y *Ax* = *T*, *dxi*, and *dxi* + *Axd* = the Angle the Plane *Ax* makes with *xi*, the Tangent to the Point *x*, then (*Theo.* 2.) *R* : *S*, *Axi* :: 68245 : 63178 the Force required.

Note, If the Surfaee of an Object at *x*, were in any given Poſition, by finding *dxi*, the Angle that the Object wou'd be ſtruck at is found, and conſequently, by the laſt Analogy, the Force required.

CASE XLI.

The Impetus Af (*Fig.* 21.) $= 4000$, and Height of an Object $Yx = 2000$, whoſe Surface reclines directly from the Perpendicular 32°, Quere the Direction and horizontal Diſtance whereat that Object may be ſtruck with the greateſt Force poſſible.

$Af - xY = 2000 = fx$ the Impetus at x, (*Theo.* 10.) then $R : S, 2\overline{|32°} :: 2000 : dx = 1798$, and becauſe the Surface of the Object reclines 32°, the Angle $d\,x\,i$, which the Ordinate and Tangent to the Point x make, muſt be 32°, and (*per Theo.* 9.) $fxd = 90° - 2\overline{|dxi}$, therefore $R : \Sigma, 2\overline{|32°} :: 2000 : 877 = df$, and $dC (= xY) - df = Cf = 1123$, then $Af : Cf :: R : S, fAC = 16°. 18'$, and $45° + \frac{1}{2} fAC = 53°. 09'$ the Direction required: Laſtly $R : S, AfC :: Af : AC = 3839$, and $AC + CY (= dx) = AY = 5637$ the Diſtance ſought.

CHAP.

CHAP. III.

Of RANDONS made on Descending Planes, &c.

CASE XLII.

THE greateſt horizontal Randon of a Piece being 8000 = 2 *Af*, (*Fig.* 12.) Quere her greateſt Randon on a Plane whoſe Deſcent is Y*AX*= 13°.

R – *S*, Y*AX*: *R* :: 2 *Af*: *AX*= 10322 the Randon ſought. (*Cor.* 8. *Theo.* 12.)

CASE XLIII.

The gr. Ran. on a Plane whoſe Deſcent is 13° being *AX* = 10322, Quere

Quere her greateſt Randon on the Plane of the Horizon.

$R : R - S, 13° :: AX : 2Af = 8000$ the Randon ſought.

CASE XLIV.

The greateſt Randon on a Plane whoſe Deſcent is 13° being 10322, Quere the gr. Ran. on a Plane whoſe Deſcent is 6°. 30′.

$R - S, 6°. 30' : R - S, 13° :: 10322 : 9021$ the Ran. ſought. (*Cor.* 5 and 8. *Theo.* 12.)

CASE XLV.

The gr. Ran. on a Plane whoſe Deſcent is 13° being 10322, Quere the gr. Ran. on a Plane whoſe Aſcent is 6°. 30′.

$R + S, 6°. 30' : R - S, 13° :: 10322 : 7186\frac{1}{2}$ the Ran. required. (*Cor.* 5 and 8. *Theo.* 12.)

CASE XLVI.

A Piece whoſe gr. Ran. on the Plane of the Horizon is 8000, being planted at *A*, (*Fig.* 21.) whoſe Height above the Plane of the Horizon is $AN =$ 2322, Quere her gr. Ran. *NX* from that Height, and the Direction to reach it.

$8000 + AN = AX = 10322$. (*Cor.* 7. *Theo.* 12.) and (by 47. *El.* 1.) $\sqrt{AX^q - AN^q} = NX = 10057\frac{2}{5}$ the Randon ſought, then $AX : NX :: R : S$, $NAX = 76°. 58'$, and $\frac{1}{2} NAX = 38°. 29'$ the Direction ſought. (*Cor.* 2. *Theo.* 12.)

CASE XLVII.

A Piece whoſe Randon on the Plane of the Horizon at $64°. 02' = MAt$ (*Fig.* 21.) is $AM = 6060$, Quere her Ran.

Ran. with the ſame Elevation on a Plane whoſe Deſcent is $MAX = 13°$.

$T, MAt + T, AXR = T, 66°.\ 22' = T, RXM$, (*Cor.* 1. *Theo.* 14.) & $RXM - AXR = AXM = 53°.\ 22'$, whence $AMX = 113°.\ 38'$, and $S, AXM : S, AMX :: AM : AX = 6919$ the Randon ſought.

CASE XLVIII.

A Piece whoſe Randon at $64°.\ 02' = MAt$, (*Fig.* 21.) on a Plane whoſe Deſcent is $MAX = 13°$, being $AX = 6919$, Quere her Ran. on the Plane of the Horizon at the ſame Elevation.

$T, MAt + T, MAX = T, 66°.\ 22' = T, RXM$, (*Cor.* 1. *Theo.* 14.) and $RXM - AXR = AXM = 53°.\ 22'$, whence $AMX = 113°.\ 38'$, then $S, AMX : S, AXM :: AX : AM = 6060$, the Randon ſought.

CASE XLIX.

A Piece whoſe Randon on the Plane of the Horizon at 40°. is 7580, Quere her Randon at 64°. 02, on a Plane whoſe Deſcent is *MAX* (*Fig*. 21.) = 13°.

S, 2 $\overline{40°}$: *S*, 2 $\overline{64°.02'}$ :: 7580 : 6060 = *AM*, (*Cor*. 6. *Theo*. 10.) then *T*, 64°. 02′ + *T*, 13° = *T*, 66°. 22′ = *T*, *RXM*, and *RXM* − *RXA* = *AXM* = 53°. 22′, whence *AMX* = 113°. 38′, and *S*, *AXM* : *S*, *AMX* :: *AM* : *AX* = 6919 the Randon ſought.

CASE L.

A Piece whoſe Randon at 38°. 30′ = *MAI*, (*Fig*. 22.) on a Plane whoſe Deſcent is *MAD* = 13° being *AD* = 1124, Qnere her Randon on the ſame Plane at 10°. 35′.

T, *MAI* + *T*, *MAD* = *T*, 45°. 44′, whence the Angles at *D* and *M* are

32°.

32°. 44′ and 134°. 16, (*Cor.* 1. *Theo.* 14.) then *S*, *AMD* : *S*, *ADM* : : 1124 : 849 = *AM*, the horizontal Ran. at 38°. 30′, and (*per Cor.* 6. *Theo.* 10.) $S, 2\,|\overline{38^\circ.\,30'}$: $S, 2\,|\overline{10^\circ.\,35'}$: : *AM* : *Am* = 315, the Randon at 10°. 35′, whence (*per Cor.* 1. *Theo.* 14.) the Angles at *d* and *m*, are 9°. 31′ and 157°. 29′, and *S*, *Adm* : *S*, *Amd* : : *Am* : *Ad* = 730 the Ran. required.

CASE LI.

A Piece whose Ran. at 10°. 35′ on a Plane whose Descent is 13° = *MAd* (*Fig.* 22.) is *Ad* = 730, Quere her Ran. at 41°. 45′ on a Plane whose Descent is *MAx* = 6°. 30′.

T, 10°. 35′ + *T*, 13° = *T*, 22°. 31′, whence the Angles at *d* and *m* are 9°. 31′ and 157°. 29′, (*Cor.* 1. *Th.* 14.) then *S*, *Amd* : *S*, *Adm* : : *Ad* : *Am* = 315, and (*per Cor.* 6. *Theo.* 10.) $S, 2\,|\overline{10^\circ.\,35'}$: $S, 2\,|\overline{41^\circ.\,45'}$: : *Am* : *AM* = 866 = the horizontal

rizontal Randon at 41°. 45′, then *T*, 41°. 45′ + *T*, 6°. 30′ = *T*, 45°. 11′, whence the Angles at *X* and *M* are 38°. 41′ and 134°. 49′, laſtly *S*, 38°. 41′ : *S*, 134°. 49′ :: 866 : 983 the Randon ſought.

CASE LII.

A Piece whoſe horizontal Ran. at 38°. 30′ is *AM* (*Fig.* 21.) = 849, being planted at *A* with the ſame Elevation ſtrikes an Object at X whoſe horizontal Diſtance is *NX* = 1095, Quere *AN* the perpendicular Height of the Piece.

NX − *AM* = 246 = *RN*, then *T*, 38°. 30′ : *R* :: *AM* : 1067 = *L*, (*Lem.* 2.) and *L* : *NX* :: *RN* : *NA* = 252½ *fere*, the Height required.

CASE LIII.

A Piece planted at *A* (*Fig.* 21.) whoſe perpendicular Height above the Plane of of the Horizon is $AN = 252\frac{1}{2}$, and there elevated $38^{\circ}.\ 30'$, ſtrikes an Object at X, whoſe horizontal Diſtance is $NX = 1095$, Quere her Ran. on the Plane of the Horizon with the ſame Elevation.

$NX : NA :: R : T, AXN = T, 13^{\circ}$, and $T, 13^{\circ} + T, 38^{\circ}.\ 30' = T, 45^{\circ}.\ 44'$ $= T, ARN$, (*Cor.* 1. *Theo.* 14.) then $S, ARN : S, RAN :: AN : RN = 246$, and $NX - RN = 849 = AM$, the Randon required.

CASE LIV.

A Piece whoſe Randon on the Plane of the Horizon at $38^{\circ}.\ 30'$ is $AM = 894$, (*Fig.* 21.) being planted at *A* whoſe Height above the Plane of the Horizon is $AN = 252\frac{1}{2}$, Quere how far ſhe will carry from that Height with that Direction.

R:

$R:T, 38°. 30' :: \frac{1}{4} AM : CV = 177$, (*Cor.* 2. *Theo.* 9.) and $CV + AN = VP = 429\frac{1}{2}$, then (*Lem.* 2.) $T, 38°. 30' : R :: AM : L = 1067$, *fere*, and $\sqrt{L \times VP} = 677 = PX$, (*Theo.* 7.) laſtly $\frac{1}{2} AM + PX = NX = 1124$ the Diſtance required.

CASE LV.

A Piece whoſe Impetus is 4000, being planted at *A*, (*Fig.* 5.) whoſe Height above the Plane of the Horizon is $AP = 624$, Quere how far ſhe will carry with a horizontal Direction from that Height.

Thus $4 \times 4000 = 16000 = L$, (*Cor.* 3. *Theo.* 7.) and $\sqrt{L \times AP} = 3160 = PK$ the Diſtance ſought.

CASE LVI.

A Piece whoſe gr. Ran. on the Plane of the Horizon is 8000, being planted at at *A*, (*Fig.* 16.) whoſe Height above the Plane of the Horizon is $AN = 188$, and there depreſs'd below the Horizon 7°. 30′, Quere how far ſhe will carry from that Height with that Direction.

$\frac{1}{2}$ gr. Ran. $= 4000 = Af$ the Impetus, and (*Theo.* 8.) $Rf = Af + AN = 4188$, alſo, (by *Cor.* 12. *Theo.* 12.) $fAN = AfC =$ double the Depreſſion, then $R : S, AfC :: Af : AC = 1036$, and $R : S, fAC :: Af : fC = 3864$, alſo $fC - AN = Pf$, and $\sqrt{Rf^q - Pf^q} = PR$, laſtly $PR - AC = RN = 970\frac{1}{2}$ the Diſtance required.

CASE LVII.

A Piece planted at *A* (*Fig.* 16.) whoſe Height above the Plane of the Horizon is $AN = 188$, and there depreſs'd 7°. 30′, lays her Shot at *R*, whoſe horizontal Diſtance is $RN = 970\frac{1}{2}$, Quere her Randon on the Plane of the Horizon at an Elevation of 38°. 30′.

RN : *AN* : : *R* : *T*, *ARN*=*T*, 10°. 58′, and (*Cor.* 1. *Theo.* 14.) *T*, *ARN*−*T*, 7°. 30′ = *T*, 3°. 33′, whence *QAN* = 86°. 27′, then *S*, *AQN* : *S*, *QAN* : : *AN* : *QN* = 3049, and *QN*−*RN* = *Am* = 2078½; lastly *S*, 2|7°. 30′ : *S*, 2|38°. 30′ : : *Am* : 7825 the Distance required.

CASE LVIII.

A Piece whose Impetus is *Af* (*Fig.* 20.) = 4000, Quere her Direction to strike an Object whose Distance is *AX* = 7450, on a Plane whose Descent is Y*AX* = 6°. 30′.

R : *S*, Y*AX* : : *AX* : XY = 844, and *Af*+*X*Y = *f*X, (*Cor.* 2. *Theo.* 8.) then *AX* : 2 *Af*+*X*Y : : XY : *XQ* = 1002, and $\frac{AX-XQ}{2} = AP = 3224$, also *Af* : *AP* : : *R* : *S*, *AfP* = 53°. 42′, and

and $AfP+Pfd=Afd=60^\circ.12'$, laſtly $90^\circ-\frac{1}{2}Afd=59^\circ.54'$ = the higher, and $\frac{AfP-Pfd}{2}=23^\circ.36'$ = the lower Elevation. (*Cor.* 12. *Theo.* 12.)

Note, If the higher Direction had been greater than the Complement of the Deſcent of the Plane, the lower wou'd have been a Depreſſion, which is plain from the *Figure*.

CASE LIX.

A Piece whoſe Randon at $63^\circ.16'$, on a Plane whoſe Deſcent is MAX, (*Fig.* 21.) $=6^\circ.30'$, is $AX=7450$, Quere her Direction to ſtrike an Object on the Plane of the Horizon whoſe Diſtance is 6000.

$T,63^\circ.16'+T,6^\circ.30'=T,64^\circ.32'$, whence (*Cor.* 1. *Theo.* 14.) $AXM=64^\circ.32'-6^\circ.30'=58^\circ.02'$, and $AMX=115^\circ.28'$; then $S,AMX:S,AXM$

 $::AX$

:: $AX : AM$=7000, and 7000 : 6009 :: S, 2 | 63°. 16′ : S, 43°. 32′, half of which is 21°. 46′ the Direction sought. (*Cor.* 6. *Theo.* 10.)

CASE LX.

A Piece whose Impetus is Af, (*Fig.* 20.) = 4000, Quere her Direction to strike an Object at X, whose horizontal Distance is AY = 7402, and whose Depth below the horizontal Line is YX = 844.

$AY : YX :: R : T, YAX = T$, 6°. 30′, and $S, YAX : R :: YX : XA$ = 7450, then (*per Cor.* 12. *Theo.* 12.) $AX : 2Af + XY :: XY : XQ$ = 1002, and $\frac{AX - XQ}{2} = AP$ = 3224, also $Af : AP :: R : S, AfP$ = 53°. 42′, and $AfP + Pfd = Afd$, lastly $90° - \frac{1}{2}Afd$ = 59°. 54′ = the Direction required.

CASE LXI.

A Piece whoſe horizontal Amplitude is *AM* (*Fig.* 21.) = 7000, ſtrikes an Object with the ſame Direction whoſe Diſtance is *A*X = 7450, on a Plane whoſe Deſcent is *MAX* = 6°. 30′, Quere that Direction.

Since 6°. 30′ is the Angle included, 86°. 45′ is half the Sum of the other two, and $AX + AM : AX - AM ::$ *T*, 86°. 45′ : *T*, 28°. 43′, whence 86°. 45′ − 28°. 43′ = 58°. 02′ = *AXM*, and *AXM* + *M AX* = *RXM* = 64°. 32′, then (*Cor.* 1. *Theo.* 14.) *T*, *RXM* − *T*, *R*X*A* = *T*, 63°. 16′ the Direction ſought.

CASE LXII.

A Piece planted at *A* (*Fig.* 21.) whoſe Height above the Plane of the Horizon

zon is $AN=844$, and elevated $20°.14'$, ſtrikes an Object at X whoſe horizontal Diſtance is $NX=7402$, Quere her gr. Ran. on the Plane of the Horizon from that Height.

$NX:NA::R:T,NXA=T,6°.30'$, and $T,NXA+T,20°.14'=T,25°.46'=T,ARN$; (*Cor.* 1. *Theo.* 14.) then $S,ARN:S,RAN::AN:RN=1748$, and $NX-RN=AM=5654$, alſo $S,2\overline{|20°.14'}:R::AM:8712=$ the gr. Ran. (*Cor.* 6 and 7. *Theo.* 10.) and gr. Ran. $+AN=9556=AX$ (*Fig.* 12.) = the greateſt Ran. on the Plane AX, (*Cor.* 7. *Th.* 12.) and $\sqrt{AX^q-AN^q}=NX=9518\frac{1}{2}$ the Randon ſought.

CASE LXIII.

A Piece planted at A, (*Fig.* 21.) whoſe Height above the Plane of the Horizon is $AN=844$, and there elevated $20°.14'$ ſtrikes an Object at X, whoſe horizontal

zontal Diſtance is $NX = 7402$, Quere her greateſt Ran. on a Plane whoſe Deſcent is $13°$.

$NX : NA :: R : T, NXA = T, 6°. 30'$, and $T, NXA + T, 20°. 14' = T, ARN$; (*Cor.* 1. *Th.* 14.) then $S, ARN : S, RAN :: AN : RN = 1748$, and $NX - RN = AM = 5654$, (*per Fig.*) and $S, 2\overline{|20°. 14'} : R :: AM : 8712 =$ gr. Ran. on the Plane of the Horizon; (*Cor.* 6 and 7. *Th.* 10.) laſtly (*Cor.* 8. *Th.* 12.) $R - S, 13° : R :: 8712 : 11240$ the Randon required.

CASE LXIV.

A Piece planted at *A*, (*Fig.* 21.) whoſe Height above the Plane of the Horizon is $AN = 844$, and there elevated $20°. 14'$ ſtrikes an Object at X, whoſe horizontal Diſtance is $NX = 7402$, being planted on the Plane of the Horizon, Quere how far ſhe can ſtrike an Ob-

jeƈt

ject on the Top of a Perpendicular YX (*Fig.* 11.) = 886.

$NX : NA :: R : T, NXA = T, 6°. 30'$, and $T, NXA + T, 20°. 14' = T, ARN$ (*Cor.* 1. *Theo.* 14.) $= T, 25°. 46'$; then $S, ARN : S, RAN :: AN : RN = 1784$, and $NX - RN = AM = 5654$, alſo (*per Cor.* 6 and 7. *Theo.* 10.) $S, 2\overline{|20°. 14'} : R :: AM : 8712 =$ gr. Ran. & gr. Ran. $- YX = 7826$ (*Fig.* 11.) $= AX$, (*Cor.* 3. *Th.* 12.) laſtly $\sqrt{AX^q - YX^q} = AY = 7775.7$ the Diſtance required.

CASE LXV.

A Piece whoſe gr. Randon on the Plane of the Horizon is 8000, Quere her Direction and Diſtance, on a Plane whoſe Deſcent is 6°. 30′, to ſtrike an Object at X, (*Fig.* 21.) whoſe Surface reclines directly from the Perpendicular 65°. 41′, with the greateſt Force poſſible.

Since

Since the Surface of the Object reclines 65°. 41′, the Angle *NXT*, made by the Tangent to the Point X and the horizontal Line *N*X, must be 65°. 41′, because the Stroke must be perpendicular; then *T*, NX*T* − 2*T*, *AX*N = *T*, 63°. 16′, the Elevation, (*Theo.* 14.) and *T*, 63°. 16′ + *T*, *A*XN = *T*, N*XM* = *T*, 64°. 32′, whence the Angles *A*X*M* and *AM*X are 58°. 02′, and 115°. 28′; then *R* : *S*, 2|63°. 16′ :: gr. Ran. : *AM* = 6428, (*Cor.* 6. *Theo.* 10.) and *S*, *AXM* : *S*, *AMX* :: *AM* : *AX* = 6841 the Distance sought.

CASE LXVI.

The Impetus of a Piece being *Af* = Y*K* (*Fig.* 12.) = 4000, and the Weight of her Shot 63 Pounds, Quere the absolute Force at the Point X, whose Distance is *AX* = 6841, on a Plane whose Descent is Y*AX* = 6°. 30′.

$R : S, YAX :: AX : XY = 774$, and $XY + KY = 4774$ = the Impetus at the Point X (*Cor.* 3. *Theo.* 10.) in Yards, which gives 14322 Feet, and (*per Prob.* 13.) $10.4 \times 63 \times \sqrt{14322} = 78410.8$ the absolute Force in Pounds required.

CASE LXVII.

The Absolute Force at the Point X (*Fig.* 21.) = 78410.8 and Direction $MAt = 63°. 16'$, together with the Descent of the Plane $MAX = 6°. 30'$ being given, Quere the Force wherewith the Plane AX, or the Surface of any Object of a given Inclination may be struck at the Point X, by the same Direction and Impetus.

$T, MAt + 2T, MAX = T, 65°. 41' = T, NXT$ (*Theo.* 14.) and $NXT - MAX = AXT = 59°. 11'$ = the Angle the Plane AX makes with XT, the Tangent

gent to the Point *X*; then $R : S$, 59°. 11′ :: 78410.8 : 67340.8 the Force required.

Note, If the Surface of the Object to be ſtruck were in any other given Poſition, by finding the Angle *NXT*, the Angle that it wou'd be ſtruck at is found, and by the laſt Analogy the Force required.

CASE LXVIII.

The greateſt Impetus of a Piece being 4000, and the Requiſite of Powder, for that Impetus, 27 Pounds, Quere the leaſt Impetus and Requiſite of Powder whereby that Piece can ſtrike an Object at X, (*Fig.* 20.) whoſe Diſtance is $AX = 900$, on a Plane whoſe Deſcent is $MAX = 13°$.

$R : S, Y\,AX :: AX : XY = 202$, and $\frac{AX - XY}{2} = 349$ = the leaſt Impetus

 required;

required; (*Cor.* 7. *Theo.* 12.) then $\sqrt{4000} : \sqrt{349} :: \overset{lb}{27} : \overset{lb}{8}$ nearly, the Charge required. (*Prob.* 11.)

CASE LXIX.

A Piece whose Impetus with 27 Pounds of Powder is 4000, being planted at *A* (*Fig.* 21.) whose Height above the Plane of the Horizon is *AN* = 202, Quere her Direction and least Charge of Powder that can reach an Object at *X*, whose horizontal Distance is *NX* = 877.

AX : NX : : *R* : *T*, N*AX* = *T*, 77°, half of which is 38°. 30′ the Elevation required, (*Cor.* 2. *Th.* 12.) and *S*, N*AX* : *R* : : N*X* : *AX* = 900, also $\frac{AX-AN}{2}$ = 349 the least Impetus; (*Cor.* 7. *Theo.* 12.) then $\sqrt{4000} : \sqrt{349} :: \overset{lb}{27} : \overset{lb}{8}$ the Charge required.

CASE

CASE LXX.

A Piece whoſe Impetus with 27 Pounds of Powder is 4000, Quere her leaſt Impetus and Requiſite of Powder to ſtrike an Object at *X*, (*Fig.* 19.) whoſe Diſtance is $AX = 1600$, on a Plane whoſe Aſcent is $YAX = 6°. 30'$.

$R : S, YAX :: AX : XY = 182$, and $\frac{AX + XY}{2} = 891$ = the leaſt Impetus; (*Cor.* 3. *Theo.* 12.) then $\sqrt{4000} : \sqrt{891} :: 27 : 12\frac{3}{4}$ (lb lb) the Charge required.

CASE LXXI.

A Piece whoſe Impetus with 27 Pounds of Powder is 4000, Quere her Direction and leaſt Charge of Powder that can ſtrike an Object at *X*, (*Fig.* 19.) whoſe Height above the Plane of the

the Horizon is $YX = 182$, and whose horizontal Distance is $AY = 1590$.

$AY : XY :: R : T, YAX = T, 6°. 30'$, and $45° + \frac{1}{2} YAX = 48°. 15'$ = the Direction; (*Cor.* 2. *Theo.* 12.) then *S*, $YAX : R :: YX : AX = 1600$, and $\frac{AX + XY}{2} = 891$ = the least Impetus, (*Cor.* 3. *Theo.* 12.) also $\sqrt{4000} : \sqrt{891} ::$ ℔ 27 : ℔ $12\frac{3}{4}$ the Charge required.

SCHOLIUM IV.

These Rules for finding the Direction and adjusting the Requisites of Gun-Powder to strike an Object with the least Impetus possible, are very useful for saving Ammunition, especially in throwing of Bums, which are not so often design'd to do Damage by their projectile Force as by the breaking of their Shells, setting Houses on Fire, and the like; but because these Effects are

are often prevented by not proportioning the Fuſe to the Time of the Flight, it will be proper to give ſome Rules for computing thoſe Times. Now, (by *Cor.* 2. *Theo.* 7.) the Time of the Flight of any Projection made on the Plane of the Horizon, is double the Time of the Fall of a heavy Body from the Height of that Projection; and the Times of the Falls from different Heights, being (by *Cor.* 2. *Theo.* 5.) in the ſubduplicate Ratio of thoſe Heights, and alſo the Fall in one Second of Time, being, in round Numbers, 16 Feet, if h = the Height of any Projection made on the Plane of the Horizon, it will be $\sqrt{16} : 1'' :: \sqrt{h} : \frac{\sqrt{h}}{4}$, equal the Time of the Fall from the Height of the Projection, and $\frac{1}{2}\sqrt{h}$ = the Time of the Flight of the Shot.

CASE

CASE LXXII.

The greatest Randon of a Piece being AM, (*Fig.* 13.) = 25600 Feet, Quere the Time of the Flight of her Shot.

$\frac{1}{4} AM = 6400 = Fv$ the Height of the Projection; because $Fv = vT = \frac{1}{2} AF$, (*Cor.* 1 and 2. *Theo.* 9.) and (*per Scholium* 4.) $\frac{1}{2}\sqrt{6400} = 40''$ the Time required in second Minutes.

Note, If the Time of the Flight at any Elevation be counted by Vibrations of a Pendulum of any convenient Length, the Time of the Flight at any other Elevation is had. (*Theo.* 11.)

CASE LXXIII.

The Time of the Flight of a Shot made at 45°, on the Plane of the Horizon being 40 Vibrations, Quere the Time at 7°. 13′.

S, 45°.

$S, 45^\circ : S, 7^\circ. 13' :: 40 : 7$ the Vibrations required.

CASE LXXIV.

A Piece whoſe Randon at $52^\circ. 30'$ is $AM = 4356$, (*Fig.* 19.) the Time of the Flight of her Shot being 24 Vibrations, Quere the Time of its Flight to X, whoſe horizontal Diſtance is $AY = 3446$.

$Am : AY :: 24 : 19$ the Time required.

CASE LXXV.

A Piece whoſe horizontal Randon at $52^\circ. 30'$ is Am (*Fig.* 16.) $= 4356$, the Time of the Flight of her Shot being 24 Vibrations, being planted at *A*, above the Plane of the Horizon, with the ſame Direction placeth her Shot at *Q*, whoſe horizontal Diſtance from the Perpendicular *AN* is *NQ* = *AM* = 5808, Quere the Time of the Flight to *Q*.

Am : *AM* : : 24 : 32 the Time required. (*Cor.* 3. *Theo.* 6.)

CASE LXXVI.

A Piece whoſe horizontal Randon at 52°. 30′ = *mAL*, is *Am* (*Fig.* 16.) = 4356, the Time of the Flight of her Shot being 24 Vibrations; the ſame Piece being planted at *A*, (*Fig.* 5.) with a horizontal Direction *AI*, placeth her Shot at *K*, below the horizontal Line, whoſe horizontal Diſtance is *PK* = *AI* = 4198, Quere the Time of the Flight from *A* to *Q*.

90°. − *mAL* (*Fig.* 16.) = *ALm* = 37°. 30′, then *S*, 37°. 30′ : *R* : : *Am* : *AL* = 7156, and (*per Cor.* 4. *Theo.* 6.) *AL* (*Fig.* 16.) : *AI* (*Fig.* 5.) : : 24 : 14 the Time required.

CASE

CASE LXXVII.

A Piece whoſe horizontal Randon at 7°. 13′ is 1086, the Time of the Flight of her Shot being 4 Vibrations; the ſame Piece being planted at *A*, (*Fig.* 16.) above the Plane of the Horizon, and there depreſs'd 7°. 13′ placeth her Shot at *R*, whoſe horizontal Diſtance from the Perpendicular is *RN* = 814, Quere the Time of the Flight of her Shot from *A* to *R*.

1086 : 814 :: 4 : 3 the Time required.

PART. III.

The *Description* and *Use* of a new *Mathematical Instrument.*

HAVING in the foregoing Pages given the Demonstration and Solution, by Calculation, of all the Cases, I think requisite in the Projectile Part of Gunnery, I shall now proceed to give the Description and Use of an Instrument, which, by placing its Parts in certain Positions, will solve any Problem before mentioned by Inspection only; as also, with a small Addition, measure the Height, or Distance of an Object on any Plane accessible, or inaccessible, together with the Ascents, or Descents of those Planes.

Tho' Instrumental Solutions are not so exact as those done by Calculation, if this Instrument is accurately made, the Ease and Expedition of it will more than recompence a small Fraction of an Error, especially to the Practical Ingeneer, who, in the Heat of Action, may often have Occasion for new Directions, and several other Things which he has not leisure to find by Calculation.

Let

Let a Brass Semicircle (*Fig.* 26.) of about nine Inches Radius, have its inner Edge, or Limb, divided into 90 equal Parts, beginning at *N* and counting upwards 10, 20, 30, *&c.* to 90, at *Z*, and each of those Divisions subdivided into 6 equal Parts. Let the outer Limb be divided into Degrees and 6th. Parts of a Degree, marking the Degrees from the Middle of the Limb both Ways 10, 20, 30, *&c.* to 90 at *N* and *Z*. Let also the middle Space, between the outer and inner Limbs, be mark'd from *Z* to *N*, 10, 20, 30, 40, *&c.* to 180 at *N*.

Let this Semicircle be fixt to the Middle of a Box Ruler *BD*, about 3½ Foot long, an Inch and half broad, and of a convenient Thickness. The inner Half of the Breadth of this Ruler must be level with the Surface of the Semicircle, but the outer Half must be higher about $\frac{2}{10}$ of an Inch. On the

outer

outer Half there must be fixt a thin Brass Scale of an equal Length and Breadth with the Box Ruler, the Breadth of which Scale is to be divided, by Lines drawn from End to End, into three equal Parts, and the Length into Inches, half Inches, and 10th. of an Inch; the Inches are to be drawn directly cross the whole Breadth, and mark'd 1. 2. 3. 4. *&c.* both Ways to *B* and *D*; the half Inches are to be drawn cross the middle and innermost Third, and the 10ths. only cross the inner Third. Let there be on one End of this Scale an Inch, and on the other End half an Inch, each divided very exactly into ten equal Parts Diagonal Ways, that the Tenths and Centesms which may happen in the Operations, on the Square and Indices hereafter to be describ'd, may be exactly measur'd on them by a Pair of Compasses. The Reason for raising the outer Half of the Box Ruler, above the inner Half, two Tenths of an Inch, as before mentioned, is to make room for the Indices *Ab* and *Ad*, which are to be fixt to the Center of the Semicircle, and there to open and ſhut, as Occaſion requires, like the Legs of a Sector. Thoſe Indices are to be about 26 Inches long, $\frac{3}{4}$ of an Inch broad, and about $\frac{2}{10}$ thick; their Breadth is to be divided into three equal Parts, and their Length into Inches, half Inches and Tenths, as the Braſs Scale before mention'd: the Inches are to be mark'd from the Center *A*, 1. 2. 3. 4. *&c.* to *b* and *d*, and the Tenths drawn croſs the inner Third. Each of thoſe Indices muſt have a ſmall Screw Nut with a Pin, or Bit of Wire upon it, which Pin may, by the Screw Nut, be fixt exactly to any Diviſion on them in order to ſuſpend the Label, or Ruler *T*Y, which has a thin Piece of Braſs with a ſmall Hole in it, exactly fitting the foreſaid Pin, and is to be fixt alſo to any Diviſion of the Ruler as Occaſion requires. Let this Label, or Ruler, be about two Foot long, and of the ſame Breadth and Thickneſs with the Indices *Ab* and *Ad*, and divided after the ſame Manner as they are, only the Tenths are to be drawn croſs the inner Edge as well as croſs the inner Third of the Breadth, and the Inches are to be mark'd 1. 2. 3. 4. &c. from *C* to *T*, and Y, making *CT* eighteen Inches, and *C*Y ſix. The like Diviſions are to be made on the Side of the Square *K*X, beginning at the inner Edge of the Braſs Ruler at *K*, marking the full Inches on the upper Side 1. 2. 3. 4. &c. to 24, the Tenths are to be drawn croſs the upper Third and the upper Edge. Let this Inſtrument be fixt upon Feet with a Ball and Socket like thoſe of a common ſurveying Inſtrument, but ſtronger in order to keep it very firm; and let there be Sights which may, as Occaſion require, be fixt on the Diameter, Indices and Ruler *T*Y, of the ſame Kind with thoſe of the

Surveying

Surveying Instrument before mentioned, which are too well known to want a farther Description.

Note, the Ball and Socket must not be fixt exactly under the Center of the Semicircle, but some Distance from it, on the Cross-Bar which goes from the Center to the Middle of the Limb, as well to support the Head of the Instrument more easily, by being nearer its Center of Gravity, as to make room for an Air-level made with a Glass Tube and Spirits of Wine, which must be fixt exactly under the Diameter, or Ruler *AB*, so that when the Semicircle is turn'd vertically the Diameter may be fixt in a horizontal Position.

The Instrument being thus prepar'd let there be a Chain of 100 Links marked with double Rings at each 10 Links, in order to have the Fractions which may happen in measuring any Distance in Tenths and Centesms, agreeing with the Diagonal Scale before mentioned: this Chain may be of any convenient Length, but because most of the new *English* Pieces of Ordnance have their greatest horizontal Randons known, or engraven on them in Paces, or Yards, in my Oppinion, 100 Foot will be a convenient Length, for if every Chain be reckon'd ten Integers if the gr. Ran. of any Piece is multiply'd by 3, and one Figure cut off for a Decimal, you will have that Randon in Tenths of your Chain, or in Integers, each of which is ten Foot.

E. g. The gr. Ran. of a Cannon Royal is, commonly, computed to be 8000 Yards, this multiply'd by 3 and a Cypher cut off; gives 2400 now two Foot, or 24 Inches on the Index of your Instrument, being taken for this 2400, $\frac{1}{100}$ of an Inch, which is very distinguishable on the Diagonal Scale, is 10 Foot on the Ground, or an Inte-

ger

ger of the foresaid 2400. The Ingenious Practitioner may find out some useful Alterations in both the Length and Division of the Parts of the Instrument and Chain; to such I leave the farther Consideration of it; and shall next proceed to

THE USE *of the* INSTRUMENT *in* LONGIMETRY, *or measuring* HEIGHTS *and* DISTANCES.

EXAMPLE I.

LET it be required to find the Distance from the House at *A*, to the Castle, (*Fig.* 24.) or to any Part thereof as the Weathercock on the Top of the Spire at *C*.

Having set up your Instrument at *A*, turn it about till through the Sights on the Diameter you ſpy a Mark ſet up at *B*, in any convenient Place of the Field about you, and having fix'd the Diameter in that Poſition, turn the moving Index till through the narrow Slit of a ſmall Sight fix'd upon the Center you ſee the Hair in the other Sight cut the Spire at *C*, then fixing the Index in that Poſition to the Limb of the Semicircle, meaſure with your Chain in a ſtraight Line from *A* to *B*; and having mark'd the Chains and Links of that Diſtance on the Diameter, and plac'd the Ruler with the Sights on it exactly to that Diſtance by means of the ſmall Pin and Hole mention'd before, ſet up your Inſtrument juſt at the End of the Diſtance you meaſur'd, (which, if you pleaſe, you may take all in full Chains, if the Ground will allow it, for you need not care whether you are ſhort of the Mark *B*, or over it, provided you keep in the ſame ſtraight Line) and turn it about till through the Sights on the Diameter

ameter you ſpy a Mark ſet up at your firſt Station *A*, and having fix'd it in that Poſition, turn the Ruler on the Pin, which is fix'd at the former Diſtance on the Diameter, till through the Sights on it you ſpy the Weather-cock at *C*, then will the Part of the Index *a c*, cut by the inner Edge of the Ruler, give the Diſtance *AC* from the Houſe to the Spire at *C*, which was to be found; and if there is Occaſion the Diſtance from the Mark at *B*, to the Spire will be found on the Ruler at the Interſection of the Index; all which is plain from the Similarity of the Triangles *A B C*, and *a*, Pin, *c*, or that form'd by the Diameter, Index and Ruler, and *Cor*. 1. 4. *El*. 6.

Note, If the Diſtance from *A*, to two, three, four, &c. Objects, had been required, if when the Diameter was firſt fix'd in the Poſition *AB*, the Index had been turn'd to each of thoſe Objects and

and the Degrees cut by the Index on the Limb at the Direction to each reſpective Object noted down in a little Field Book. Thus,

Degrees.	Diſtances.	Objects.
34½		Wind Miln.
10		Tree.
25½		Buſt on the Wall.
40¾		Weathercock.

Then the Diſtance *AB* being meaſur'd, and the Inſtrument fix'd at *B*, as before directed, if the Index is plac'd ſucceſſively to the Degrees noted down in the Field Book, the Ruler directed to each reſpective Object will cut upon the Index the Diſtance of that Object from the Houſe at *A*, which may be ſet down for farther Uſe, in the Diſtance Column of the Field Book; and if there is Occaſion for the Diſtance from *B*, to each Object, or for the Diſtances of thoſe from each other, they may be eaſily found; for when you direct the Ruler to any Object the Index being at the Degrees, or Angle belonging to that Object, will cut the Diſtance from it to *B*, on the Ruler, and if one of the Indices be fix'd to the Degrees belonging to one Object, and the other Index to thoſe belonging to another Object, the Diſtance between the two Points on thoſe Indices which repreſent the Diſtances of the reſpective Objects from *A*, will give the Diſtance of thoſe Objects from each other, which may be either taken in a Pair of Compaſſes and apply'd to a Scale, or found by applying the Beginning of the Diviſions on the Ruler to one of thoſe Points, and the other Point will cut the Diſtance between the Objects upon it; but from the Angles and firſt Diſtances found in the Field, the reſt may be found by the

In-

Instrument at any Time or Place you have Occasion for them.

II.

Let it be required to find the Perpendicular Height Y *X* (*Fig.* 25.) of the Top of the Wind-Miln above the horizontal Line *H O*, as also the horizontal Distance *A*Y, of that Perpendicular.

Find the Distance *A X*, by the last *Case*, and having plac'd the Semicircle vertically in the Plane *A X* Y, and with its Diameter in the horizontal Line *H O*, (which last may be done by a Line and Plummet, or by the Air-level before mentioned) move the Square along the Diameter until it cuts the first found Distance on the Index, then will the Index cut the Height required on the Square, and the Square the Horizontal Distance on the Diameter, which was to be found.

Note,

Note, If there is any Occaſion for the Angle of Aſcent Y *AX*, the Index cuts it on the outer Limb; and if the Height of the Wind-Miln from the Foundation is required, let the Semicircle continue in the ſame Poſition, and move the Index till through its Sights you ſpy the Foundation at *f*, and the Difference of what it now cuts on the Square and what it cut before is the Height required. It may not be improper to remark here, that this Part of the Propoſition relating to the Height of the Wind-Miln wou'd require that the Diſtance from it to the Inſtrument ſhou'd be but ſmall, that it may be plainly expreſs'd on the Index in Feet, which will, at leaſt, require that every Chain of the Diſtance *AX*, be an Inch on the Index; now, the Index being but 26 Inches long, it's plain the Diſtance *AX*, ought not to exceed 26 Chains, or 2600 Feet.

III.

How to meaſure the Depth *SH*, of the Ship (*Fig.* 25.) at the Surface of the Water below the horizontal Level of the Inſtrument at *B*.

This Propoſition is perform'd after the ſame Manner the laſt was, only that it will be moſt convenient to turn the Limb of the Semicircle downwards as it appears in the *Figure* at *B*.

Theſe three Examples contain all that the practical Ingeneer has Occaſion to know relating to the Menſuration of Heights and Diſtances, whether they are acceſſible, or inacceſſible; for in either Caſe there is nothing more required but that he may ſee the Object from two convenient Stations ſufficiently diſtant from each other, on any Plane whether aſcending, deſcending, or horizontal.

THE

THE USE *of the* INSTRUMENT *in* PRACTICAL GUNNERY.

IN what follows I will not trouble the Reader with Quotations, or References to former Propositions, because the Thing is so plain and easy, and so naturally follows from what is said before, that none who tolerably understands that, can be at a loss for the Reasons of this.

And the principal Propositions a practical Ingeneer has Occasion for, being, from any two of the Impetus, Distance, and Direction given, how to find the Third; I shall give Examples in all the possible Cases of these, which will sufficiently shew the Ease and Expedition of this Instrument, and lead the apt Practitioner to the Use of it in other Propositions when Occasion requires.

Note, *Whenever the moving, or placing, of the Square is mentioned, it is suppos'd to slide along the Diameter, or Ruler, still keeping K X at Right-Angles, or Perpendicular to B D.*

PROP. I.

The greateſt horizontal Randon of a Piece being 2400, Quere her Direction to ſtrike an Object whoſe horizontal Diſtance is 1584.

Place the Diſtance on the Square to the gr. Ran. on the Index, and the latter will cut $69\frac{1}{3}$ Degrees on the inner Limb of the Semicircle, which is the Direction required.

PROP. II.

A Piece whoſe gr. Ran. on the Plane of the Horizon is 2400, Quere her Randon at $69\frac{1}{3}$ Degrees.

Fix the Index to the Direction on the inner Limb, and the greateſt Randon on it will cut 1584, on the Square the Randon ſought.

PROP.

PROP. III.

A Piece whoſe horizontal Randon at $69\frac{1}{3}$ Degrees is 1584, Quere her greateſt horizontal Randon.

Fix the Index to the Direction on the inner Limb, and the given Randon on the Square will cut on the Index 2400 the gr. Ran. required.

Theſe are the principal Propoſitions relating to Projections made on the Plane of the Horizon: But if the Height and Sublimity of the Projection were required, the Square in the laſt Poſition will cut 1803 on the Diameter, one fourth the Sum and Difference of which, and the greateſt Randon is 1051, and 149, the Height and Sublimity required.

PROP. IV.

The Impetus of a Piece being 1200, Quere her Direction to ſtrike an Ob-

ject

ject whose Distance is 1728, on a Plane whose Ascent is 6½ Degrees.

Fix the lower Index *Ad*, (*Fig.* 26.) to the Ascent of the Plane on the outer Limb of the Semicircle, then move the Square to the Distance of the Object upon it, and mark the Distance *AK* cut on the Diameter by the Square; then having suspended the Ruler at *f*, by means of the small Pin and Hole before described, making *Af* and *f* Y each equal to the given Impetus, move the Ruler *f* Y, exactly along the given Distance of the Object upon the Index at *X*, until *X* Y is equal to *AK*, then will the Index *Ab*, cut 65½ Degrees, on the inner Limb of the Semicircle the Direction required.

PROP. V.

A Piece whose Impetus is 1200, Quere her Randon at 65½ Degrees, on a Plane whose Ascent is 6½ Degrees.

Fix

Fix the lower Index to the Aſcent of the Plane on the outer Limb, and the other to the Direction on the inner Limb, making *Af*, and *f*Y, (*Fig.* 26.) each equal to the given Impetus,

Laſtly, Move the Square and Ruler (keeping their Interſection exactly to the inner Edge of the Index *Ad*) until *AK*, and *X*Y are exactly equal, then will *AX*= 1728 the Randon required.

PROP. VI.

A Piece whoſe Randon at 65½ Degrees is 1728, on a Plane whoſe Aſcent is 6½ Degrees, Quere the Impetus of that Piece.

Fix the lower Index as *per* laſt *Caſe*, and the Ruler to the given Diſtance at *X*, by means of the Pin and Hole, making *X*Y equal to *AK*; laſtly turn the Ruler on the Center *X*, and the Index *Ab*, on the Center *A*, until *Af* and *f*Y

are

are equal, and each of them will be 1200 the Impetus required.

PROP. VII.

A Piece whose greatest Randon on the Plane of the Horizon is 2400, Quere her greatest Randon on a Plane whose Ascent is 6½ Degrees.

Fix the lower Index to the Elevation of the Plane, and move the Square along the Diameter until *AK*+*AX* be equal the greatest Randon given, then will *AX* be 2156 the gr. Ran. sought.

Or thus,

Place the Square to 1000, on the Index, and it will cut 113 on the Diameter, then place 1000+113 on the Index to 1000 on the Square, and having fix'd the Index, move the Square to 2400, the given Randon on the Index, and it will cut 2156 on the Square the greatest Randon sought. *PROP.*

PROP. VIII.

A Piece whoſe greateſt horizontal Randon is 2400, Quere the greateſt horizontal Diſtance from which that Piece can ſtrike an Object on the Top of a Perpendicular whoſe Height is 400.

Place the Square to 400, on the Diameter; and 2000 on the Index (the Complement of the Height to the given gr. Ran.) will cut 1685 on the Square which is the horizontal Diſtance required.

PROP. IX.

A Piece whoſe gr. Ran. on the Plane of the Horizon is 2400, Quere the greateſt Height which that Piece can reach on a Perpendicular whoſe horizontal Diſtance is 1685.

Keep the Edge of the Index to the given horizontal Diſtance on the Square, and move the Latter until the Sum of the Numbers cut on the Diameter and Index be equal to 2400 the given gr. Ran. and thoſe on the Diameter will be 400 the Height required.

PROP. X.

A Piece whoſe Impetus is 1200, Quere her Direction to ſtrike an Object whoſe Diſtance is 2235, on a Plane whoſe Deſcent is 6½ Degrees.

Fix the lower Index to the Deſcent of the Plane, and the Ruler to the given Diſtance upon it at *X*, (*Fig.* 27.) making *CX*=*AK* on the Diameter; laſtly, make *Af*, and *fC*, each equal to the given Impetus, and the Index *Ab*, will cut on the inner Limb 59⅙ Degrees the Direction ſought.

PROP.

PROP. XI.

A Piece whoſe Impetus is 1200, Quere her Randon at $59\frac{5}{6}$ Degrees, on a Plane whoſe Deſcent is $6\frac{1}{2}$ Degrees.

Fix the lower Index to the Deſcent of the Plane, and the upper Index to the Direction on the inner Limb of the Semicircle, and having ſuſpend the Ruler at *f*, making *Af*, and *fC*, each equal to the Impetus, move the Square and Ruler, (keeping their Interſection exactly to the inner Edge of the lower Index) until *AK*, and *CX* are equal, then will *AX* be 2235 the Randon ſought.

PROP. XII.

A Piece whoſe Randon at $59\frac{5}{6}$ Degrees is 2235, on a Plane whoſe Deſcent is $6\frac{1}{2}$ Degrees, Quere the Impetus of that Piece.

Fix the Indexes as *per* last *Case*, and the Ruler to the given Distance on the lower Index making *C X* equal to *AK*, on the Diameter; lastly, turn the Ruler and upper Index until *Af*, and *fC* are equal, and each of them will be 1200 the Impetus required.

PROP. XIII.

A Piece whose Impetus is 1200, Quere her gr. Ran. on a Plane whose Descent is 6½ Degrees.

Fix the lower Index to the Descent of the Plane on the outer Limb, and the Square mov'd to 1000 upon it will cut 113, on the Diameter, the place 1000−113 (=887) on the Square to 1000, on the Index, and having fix'd the Latter, move the Square until the Index cuts 1200, the given Impetus upon it, then will the Square cut 1353, on the Index double, which is 2706 the gr. Ran. required.

PROP.

P R O P. XIV.

A Piece whoſe greateſt horizontal Randon is 2400, being planted upon an Eminence whoſe perpendicular Height above the Plane of the Horizon is 306, Quere her gr. Ran. from that Height.

Fix the Square to the given Height on the Diameter, and 2706 (the Sum of the given Height and gr. Ran.) on the Index will cut on the Square 2686 the Randon required.

Note, The Sum of the given Numbers, and the Number ſought are each too great to have the firſt two Figures in Tenths of an Inch on the Inſtrument they may therefore be taken in Tenths of a half Inch; or if the Halves of the given Numbers are taken on Inches as before, half the Number ſought will be got on the Index.

PROP.

PROP. XV.

The horizontal Distance and Height or Depth of an Object, either above or below the horizontal Level of the Piece, being given to find the least Impetus that possibly can reach that Object.

Fix the Square to the given Height or Depth on the Diameter, and move the Index to the horizontal Distance on the Square; then will half the Sum of the given Height and Distance cut on the Index in Ascents, and half their Difference in Descents, be the Impetus requir'd.

SCHOLIUM.

It is plainly demonstrated, by *Theo.* 12. that the Direction to reach an Object with the least Force possible bisects the

the Angle between the Object and Zenith, and, by the 2d. and 7th. *Corollaries* to that *Theorem*, that Force is ſuch as wou'd carry the Shot on the Plane of the Horizon, at an Elevation of 45 Degrees, a Diſtance equal to half the Sum of the perpendicular Height and Diſtance of the Object in Aſcents; and half the Difference of the perpendicular Depth and Diſtance in Deſcents; all which are found with the greateſt Eaſe imaginable by the Examples of the Uſe of the Inſtrument in *Longimetry*.

Now having that Direction and Force, together with the Force or Impetus of his Piece with any given Charge, which is found by the 6th. or 12th. *Prop.* foregoing Propoſitions, the Engineer has nothing to do with Regard to hitting an Object, but to adjuſt his Charge of Powder by *Caſe* 11. *Page* 88. and direct his Piece according to the given Elevation, which he may do

by

by the Method deſcrib'd in *Page* 79. or by Dr. *Halley*'s Method of fixing a Piece of Looking-glaſs Plate parallel to the Muzzle of the Mortar, which is ſuppos'd to be cut ſquare to the Bore, and to raiſe or depreſs the Elevation, till the Object appears reflected from the ſame Point of the Glaſs, where on a Lead Plumbet, hung from the Obſervers Eye, falls.

I do not pretend to direct Engineers in Things that are only practical, but, as Care and Diligence are as neceſſary as Knowledge in every Employment, I hope they will excuſe me for telling them that, whether they are the Beſiegers, or the Beſieged, the Diſtances, Directions, and Requiſites of Powder for reaching any particular Object, or Paſs, which their General may think proper to attack, or defend, ought all to be ready calculated beforehand, when the Nature of the thing will allow of it.

SOME

SOME *New* PROPERTIES *of* PROJECTILE CURVES.

THE following Properties are not of any material Use in the Art of Gunnery, *but as their Demonstrations depend on the foregoing Theory, I thought it not amiss to insert them here: For, beside the Satisfaction they may probably give the curious Readers, there may perhaps be some latent Use in them which time and their farther Consideration may discover.*

The 4th. Property is hinted at by the learned Dr. Halley *who shews that the utmost Limits of the Reach of every Project, made by the same Impetus, is in the Curve of a Parabola,* &c. *but he does not there observe, that a Line drawn from the Engine to any Point in those Limits shall pass through the Focus Point of the respective Parabola whose utmost Limit that Point is, which leads to a very plain and easy Method to find the Direction necessary* *to hit any Object within the Reach of any given Impetus, whether that Object be below or above the horizontal Level of the Piece.*

The rest of these Properties together with those discovered by Theo. 13 *and* 14, *and* Corollary 4*th. to* Theo. 10*th. I have never seen or heard of before, nor am I any way vain of the Discovery, tho' I insert them here as new, for he is but a small Proficient in Mathematicks that could not discover new Properties in Curves every Day of his Life, if he imploy'd his Mind that Way; yet so extensive is Science, and so limited human Capacity, that the simplest Curve wou'd afford Imployment to the greatest Genius that ever was, tho' he were to live a thousand Years.*

I.

THE Points of Interſection of the Tangent and Axis produc'd, of every Parabola that can be deſcrib'd by the ſame Impetus in the ſame vertical Plane, will be in the Circumference of a Circle whoſe Radius is the Impetus, and whoſe Center the higheſt Point of that Impetus.

By *Cor.* 1. *Theo.* 9. the diſtance fT (*Fig.* 9.) from the Focus to the Interſection of the Tangent and Axis produc'd, is ſtill equal to the Impetus Gf, or GR; and ſince the End f (*Fig.* 13.) of the Line ft is ſtill in the Circumference of a Circle, (*Cor.* 4. *Theo.* 10.) ft, being ſtill equal and parallel to it ſelf, the End t muſt alſo be in the Circumference of a Circle, of which it's Evident Z muſt be the Center.

II.

The principal Vertexes $V. v. u.$ &c. (*Fig.* 13.) of every Parabola deſcrib'd by the ſame Impetus, *&c.* is in the Periphery of an Ellipſis whoſe conjugate Diameter is the Impetus AZ, and whoſe tranſverſe Diameter is double that Impetus.

By *Cor.* 7. *Theo.* 10. 'tis evident the principal Vertex v of the Parabola AvM deſcrib'd by an Elevation of 45 Degrees is at the greateſt Diſtance poſſible from the Impetus AZ, that is pv equal and parallel to AF, $=$ AZ; and if the Direction was in the ſame Plane on the other Side, or depreſs'd on this Side 45 Degrees, the principal Vertex wou'd be in vp produc'd that Way as far diſtant from the Impetus AZ on that Side as it is on this; conſequently $2pv = 2AZ$ is the longeſt Axis of the Curve. Now becauſe $Fv =$ vT, (*Cor.* 2. *Theo.* 9.) and $VK = Cu$, (*Cor.* 9. *Theo* 10.) the Line vp, produc'd when the Vertexes fall on the other Side, will ſtill biſect the Line Vu that joyns the Vertexes of any two Parabola's made by Elevations equally above and below 45 Degrees. Thoſe Vertexes are therefore in the Periphery of an Ellipſis. *Q. E. D.*

Or thus,

Let $Af = AZ = x$, and $AC = pn = y$, (n being taken any where in the right Line pv) then will $\sqrt{xx - yy} = fC$. But becauſe $fV = Cu$, and pu biſects uV from above, $\frac{\sqrt{xx - yy}}{2} = Vn$, the Ordinate to the Point n; which Equation is expreſſive of an Ellipſis; for, in that Figure the Square of the Tranſverſe Diameter, is to the Square of the Conjugate, as the Rectangle of any two Abſciſſa's, to the Square of the Ordinate; that

is,

is, by the Subſtitution and what was ſaid above, $4xx : xx :: xx - yy : \frac{xx - yy}{4}$ which is evidently the Square of $\sqrt{\frac{xx - yy}{2}}$ the Value of nV found before. *Q. E. D.*

III.

If *C* (*Fig.* 28.) is the Place of an Engine, and *CZ* its Impetus, all the Parabola's that can be deſcrib'd by that Impetus, &c. will have the outward Extremities of their Parameters, or that pointing to the Right-hand, in the Periphery of an Ellipſis *ZqMC*, whoſe Area is equal to the Area of the Circle *ZfNS*, deſcribed on the Center *C* with the Radius *CZ*.

Let *ZM* be the Diagonal of a Square deſcrib'd on the Diameter *ZN*, equal to twice the Impetus, and let an indefinite Number of right Lines *gr*. *dn*. *Sq*, &c. be ſuppos'd to be drawn parallel to *ZB* or *NM*. By *Cor.* 4. to *Theo.* 10. all the Parabola's that can be deſcrib'd as above, will have their Focus Points in the Circumference of the Circle *ZfNS*, &c. and by *Theo.* 7. the Semi-parameters *pr*. *mn*. *fq*. &c. are ſtill equal to the correſpondent Heights *Za*. *ZK*. *ZC*. &c. or their Equals *ab*. *Kl*. *Cf*. &c. that is *ab* = *pr*.

pr. Kl=*mn. Cf*=*fq. eo*=*XV*, &c. hence, by the Doctrine of Indivisibles, the Triangles *Zab. ZKl. ZCf.* &c. are still equal to the corresponding mixt lin'd Figures *Zpr. Zmn. Zfq.* &c. and the whole mixt lin'd Figure *ZqVMNf* equal to the Triangle *ZNM*, or half the Square of the Circles Diameter Now since the Line of Direction still bisects the Angle between rhe Focus and Zenith, (*Theo.* 9.) if the Direction is horizontal the Focus Point must be in the Nadir at *N*. But if the Direction be depressed below the Horizon the Focus will be some where in the Semi-circumference *NSZ*: But still the Semi-parameters in every Point of that Semi-circumference will be equal to what they were in the other: That is at the Point *i* the Semi-parameter is *Xi*=*Ze*=*XV*, and at *S* it will be *SC*=*ZC*=*fq*, also at *d*, *dh*=*ZK* =*mn*, &c. hence the mixt lin'd Figure *ZSNMC* is equal to the other mixt lin'd Figure *ZqVMNf*, and each to the Triangle *ZNM*.

From the same Equality of the Semi-parameters *gy* and *pr*, &c. taken at any Height and their Equality to the right Line *ab*, &c. it follows that the right Lines *br*=*by*. *ln* =*lh*. *fq*=*fC*. *OV*=*OX*, &c. as also that Lines, or double Ordinates, *yr*. *hu*. *Cq*. &c. are still equal to the corresponding

Chords

Chords *gp. dm. Sf.* &c of the Circle. From the first of these Equations it follows that the Curve in which are all the outward or Right-hand Extremities of the Parameters, *&c.* is the Periphery of an Ellipsis, and by the 2d. that Ellipsis is equal to a Circle whose Radius is the Impetus of the Engine.

Q. E. D.

Beside the Equations before mention'd, I see no End to what might be found in this Figure, as that the Quadrantal Segment *Zfp* of the Circle is equal to the Elliptical Segment *ZCy*, (or *qMV*) and that each Part of those Segments cut off or lying between the same Parallels *yp. hm.* &c. are still equal. Also if right Lines are drawn from *Z* to any two corresponding Points in the Peripheries of the Circle and Ellipsis, as *Zp. Zr.* or *Zm. Zn.* the Segments cut off by those right Lines will be equal; and if right Lines were drawn from *Z* and *C* to *q* and *M*, the Parallelogram *ZqMC* wou'd be the greatest that cou'd be inscrib'd in the Ellipsis, and equal to a Square inscrib'd in the Circle; Each of the Segments cut off by the Sides of the Parallelogram wou'd be equal to the Quadrantal Segment of the Circle. As also that the Space *ZBqr* is epual to the Space *fMNX*, and *fMX*=*CXN*, &c.

IV.

If right Lines are suppos'd to be drawn from the Place of the Engine *A* (*Fig.* 13.) through the Focus Points of every Parabola that can be describ'd by the same Impetus *AZ*, and terminated by the Curve of each respective Parabola, as at *X*; those Points of Termination will all be in the Curve of a Parabola, whose Focus is *A*, Vertex *Z*, and principal Parameter four Times the Impetus *AZ*.

By *Cor* 4 *Theo.* 10. all the parabola's that can be describ'd by the Impetus *AZ* have their Focus Points in the Circumference of the Circle *AZFN*, &c.

Let $Af = x$, *f* being taken any where in that Circumference, and $fX = XR$ (*Cor.* 2. *Theo.* 8) $= y$; then will $AX = x + y$, and $YX = x - y$, but (by 47. 1. *El.*) $AX^q - YX^q = AY^q = QX^q$; (*per Figure*) that is, in Symbols, $4xy = QX^q$; but $QZ = XR = y$, and $4x = 4AZ$, therefore $4AZ \times ZQ = QX^q$, whence, and from *Theo.* 7. the Point *X* is in the Curve of a Parabola whose Focus is *A*, Vertex *Z*, and principal Parameter four Times the Impetus *AZ*.

FINIS.