An arithmetical warlike treatise named stratioticos

Digges, Thomas

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The second book.

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CHAP. I.
A briefe Treatise of that part of Algebra that concerneth Cosicall, or *Denominate Numbers.*

As in common Arithmetike al Abstract numbers take their original from the Vnite, so in these kind of concrete or Denominate numbers, we take our beginning from a *Radix* or Roote, not improperly so tearmed, sithens out of the same, as from the very base and roote are deriued infinite braunches of these Cossical numbers. And as in common numbers we procéede from the Vnite by Addition, to create all kinde of Numbers, so in this Arte of Numbers Cosicall, we procéede from the Roote by Multiplication, to create all Squares, Cubes, Zenzizenzike, and Surd Solides, with all other that in this Science are vsed, the which by Example may best be explaned.

1	2	3	4	5	6	7	8	9	10	11	12
Roo.	*Sq.*	*Cu.*	*SqS.*	*Sſo.*	*SqC.*	*BſS.*	*SSSq.*	*CC.*	*SſS.*	*CſS.*	*SSC.*
2	4	8	16	32	64	128	256	512.	1024	2048	4096

Any Number Abstract may be a Roote, and according to the value and quantitie thereof, all the other Cossicke Numbers in value & quantitie alter. For Example sake, I do here suppose the Roote 2, the which multiplied in it selfe, maketh 4 his Square, with his Character ouer his head, that *Sq.* augmented againe in his Roote, maketh 8 his Cube, and his Character ouer his head. Of these are all the rest componed. For the Square being four, againe squared, maketh his Squared square 16, with his Caracter ouer him. The next being not made by the Square or Cubike Multiplication of any of the former, cannot take his name from Square or Cube, and is therefore called a Surd solide, and is onely created by Multiplicatiõ of 2 the Roote, in 16 the *SqS.* making 32 with his cõuenient Character ouer him, & for distinctiõ is tearmed ỷ first Surd solide. Again, 8 being the *Cu.* squared, maketh 64, ỷ next number ensuing in Geometricall Progression, with his Caracter componed of *Sq.* and *Cu.* called a Squared Cube, the next being 128, is not made of Square or Cubike Multiplication of any, but only by the Multiplication of the Squared Cube in his Roote, and therefore is tearmed the B.S.solide, or second S.solide.

Heereof you may collect this Rule vniuersall, to giue name to any of these Cossike Solides: consider what simple or vncompound numbers the number of the Solide may be resolued into, and of the Characters, to those Numbers apperteining he shall be componed.

Example.

Of the sixt Cossicall number, *6* is componed of 2 and 3, 2 is the *Sq.*3 the *Cu.* I conclude therfore the sixt is a *S.Cube*, likewise of the eight, 8 is made of 2, 2 and 2 therefore I inferre, that the eight Character shal be a *S.S.Square*. Again, 10 is made of 2 and 5 a *S S*, the tenth therefore shall be a *Sq.S Solide*, and thus of all other.

This I haue rather added for custome sake, because in all parts of the world these *Characters* & names of *Sq.* and *Cu.* *&c.* are vsed, but because I find another kind of *Character* by my Father deuised, farre more readie in *Multiplications, Diuisions*, & other *Cossicall* operations, I will not doubt, hauing Reason on my side, to dissent from common custome

in

in this point.and vse these *Characters* ensuing:

ɟ ↊ ↋ ᔭ χ ↄ̸ [illegible] [illegible] [illegible].

ɟ for a *Roote*, ↊ for a *Square*, ↋ for a *Cube*, ᔭ for a *Squared Square*, χ for a *S.Solide*, ↄ̸ for a *Squared Cu.* and so of the rest, vsing onely the ordinarie *Figures*, but somewhat turned a contrarie way, bycause they should be discerned, and not confused among others, and these shall be named *Primes, Seconds, Thirds, Fourthes &c.* according to their *Figure* or *Character*.

CHAP. II.

Of Addition of Numbers Cossicall.

WHen Numbers Cossical are presented to be added, either it is of one or of mo, of one thus. I would adde 5 ɟ to 20 ɟ in this case ẏ Characters being like, you shall onely adde ẏ Numbers adioyning to the Character, so find ye that two Cossical numbers ioyned, make 25 ɟ: but if the Characters be different, as 10 ↊ to be added to 16 ↋, then shall you ioyne thē with this signe + Plus, saying they make added 10 ↊ + 16 ↋, that is to say 10 seconds more 16 thirds: for being of different Characters, they cannot be otherwise expressed, but if they be many to be added togither, thē shall you dispose thē one vnder another, matching alwayes like Characters togither. For Example, I would adde 20 ɟ + 30 ↊ + 25 ᔭ vnto 45 ɟ + 16 ↊ + 13 ↋.

In Addition of these kind of numbers, I begin frō the left hand, saying 20 and 45 make 65, whereto I adioyne their common Character ɟ. Likewise 30 and 16 make 46, I adioyne ↊ their common Character.

20 ɟ	+ 30 ↊	+ 25 ᔭ	
45 ɟ	+ 16 ↊	+ 13 ↋	
65 ɟ	+ 46 ↊	+ 25 ᔭ	+ 13 ↋

And bycause these numbers are both noted wᵗ this signe +, I adde also that Signe. Last of all, bycause 25 and 13 doe differ in Character, I may not adde their numbers, as in the other, but put them downe with the signe + as in the Example you may behold.

Againe, it happeneth sometime, that albeit the Characters agrée, yet the Signes differ, as in this secōd Example.

$$\begin{array}{l} 10\ \mathrm{I} + 25\ \mathrm{II} - 12\ \mathrm{III} \\ 13\ \mathrm{I} - 22\ \mathrm{II} + 16\ \mathrm{III} \\ \hline 23\ \mathrm{I} + 3\ \mathrm{II} + 4\ \mathrm{III} \end{array}$$

Here first 10 I and 13 I make 23 I, but bycause in the next the Signes are different, I must deduct the lesse from the bigger, so remaineth 3, to the which I adde + *viz.* the Signe of the greater number. Againe, the next hauing different Signes, I subtract the lesse out of the greater, there remaineth 4 III whereto I annex + being the Signe of the greater. And thus the Addition appeareth to be 23 Primes + 3 secondes + 4 III. Here is all the difficultie in this kinde to vse Subtraction, when the Signes differ, and set downe the Signe of the greater, the which in one short sentence is expressed by Styfelius.

Diuersa Signa commutant speciem, A ponit M.

One Example more I thinke good to adioyne, to remoue all scruple or doubt that may arise. These Operations conferred with the Rules of Addition ensuing, are manifest, and néede no further Explanation.

$$\begin{array}{l} 4\ ℞ + 25\ \mathrm{II} - 13\ \mathrm{I} \\ 6\ \mathrm{III} - 20\ \mathrm{II} + 12\ \mathrm{X} \\ \hline 4\ ℞ + 6\ \mathrm{III} + 5\ \mathrm{II} + 12\ \mathrm{X} - 13\ \mathrm{I} \end{array}$$

The Rules of Addition.

If the Signes be like, subscribe the same Signe.
If vnlike Subtract, & to the Remaine adioyne the greater Numbers Signe.

CHAP. III.

Of Subtraction.

The like different difficulties are also in Subtraction, but by one exãple cõtayning euery particular varietie that can chance, I thinke better briefly to teach, thanby vsing many wordes in euident matters to be tedious.

The rules of Subtraction.

Lke Signes produce their like, except the greater nũber be from the lesse to be subtract, then change the Signe and set downe thereto the Difference.

Contrarie *Signes* in stead of *Subtraction*, requireth *Addition*, and to the *Product* adde the vpper numbers *Signe.*

8 ℨℨ + 12 ßß	8 𝔠𝔠 — 2 ßß	— 7 ℨℨ + 10 ßß
4 ℨℨ + 10 ßß	2 𝔠𝔠 + 2 ßß	+ 5 ℨℨ + 12 ßß
4 ℨℨ + 2 ßß	6 𝔠𝔠 — 4 ßß	— 12 ℨℨ — 2 ßß

Behold in these thrée Examples ye haue as many varieties as in Subtraction can happẽ, wherein this is to be noted, that howsoeuer ỹ nũbers are deliuered, ye may alter and transpose them, so as like Characters be matched togither, allwayes kéeping with them their cõuenient Signes, and placing allwayes the number to be Subtracted vnder, and the other aboue. In the first Example we sée the Signes alike, deduct therefore one from the other, there remayneth first + 4 ℨℨ and in the second + 2 ßß. But in the second part of the sécõd Example, the Signes are different. In stead therefore of Subtracting I adde, so amounteth 4 ßß whereto I adioyne the Signe — bycause it is ỹ vpper numbers Signe. Againe in the third, bycause the Signes are diuers in the first part by Addition I finde — 12 ℨℨ. But in the secõd part albeit the Signes be one yet the greater nũber being vnder, I adioyne a contrary Signe to the difference, saying the remaine is — 2 ßß. And thus of all others.

CHAP. IIII.
Of Multiplication.

For Multiplicatiō of Coſſike numbers ye ſhall adde togither their Characters, and the reſulting Character, ſet downe with the number produced of their numbers Multiplication, and for the Signes theſe Rules enſuing muſt be obſerued.

The ſame or like Signes multiplied produce + *Plus*.
Contrarie or diuerſe Signes produce alway — *Minus*.

Euery ſeuerall number of the Multiplier muſt be augmented in euery number of the number to be multiplied, and the Productes with their correſpendent Signes and Characters ſo placed, that like Characters be matched, and finally by Addition the whole collected.

Behold the Example.

Firſt I ſay 6 times 4 makes 24 and *Primes* added to Thirdes make Fourthes. I ſet down therfore 24 ꝏ. Again 6 times 12 make 72 and *Primes* added to Seconds, make ꝗ, I ſet downe therfore 72 Thirdes, and in both theſe numbers the Signes being like, I ſet downe +

4 ꝗ + 12 ꜧ — 4 ⱡ
6 ⱡ + 12 ꝗ — 3 ꝏ

+ 48 ꝺ + 24 ꝏ + 72 ꝗ — 24 ꜧ + 144 χ — 12 ɏ
— 36 ꝺ — 48 ꝏ + 12 χ

24 ꝺ — 24 ꝏ + 72 ꝗ — 24 ꜧ + 156 χ — 12 ɏ

Againe 6 ⱡ in 4 ⱡ make 24 ꜧ, but bycauſe the Signes are differēt I adde to thoſe 24 ꜧ the Signe —. This done, I daſh 6 with the penne, and go to the next Section or mēber of my Multiplier *Viz.* 12 ꝗ ſaying 12 ꝗ in 4 ꝗ Produce 48 Sixes, I ſet downe therfore + 48 ꝺ. Againe 12 ꝗ
in

in 12 ɥ produceth 144 X the Signes being like I adioine + Againe 12 ꝣ in 4 ɥ maketh 48 ꝫ and the Signes being vnlike I set downe—. This 48 ꝫ I set directly vnder 24 ꝫ in the vpper line. Againe dashing out 12 ꝣ I say 3 ꝫ in 4 ꝣ makes 12 ℓ the which set in the vpmost line bycause there is none his like to set him vnder.

Againe 3 ꝫ in 12 ɥ maketh 36 ꝫ which I subscribe vnder the Sixtes in the vpper line adding the Signe — bycause the Signes are diuers.

Finally 3 ꝫ in 4 ɥ make 12 X the which I set vnder the Fiftes in the first row, adioyning the Signe +, bycause both their Signes be like. And so drawing a line by Addition, I finde the Product of that multiplicatiō 24 ꝫ + 156 X + 72 ꝣ — 24 ꝫ — 24 ɥ — 12 ℓ. In like sort of all other for no other change can happen that you are not instructed in by this Example.

CHAP. V.
Of Diuision.

The Diuision of numbers Cossicall is not vnlike to the Diuision of common numbers in respect of placing the Diuisor, and as in Multiplication the name to the Character resulting was giuē by adding togither the two Characters of the multiplier and multiplied number, so here the Character of the Quotient is found Subtracting the Character of the Diuisor from the Character of the Diuidend, as by the Example ensuing more plainely appeareth: the Rule of the Signes is all one in Diuision with those of Multiplication.

Like

Like Signes giue +, Vnlike —.

The Nūber diuisible.	60 X + 72 ʒ — 80 ꝣ — 96 ɥ	6 ɥ — 8 ⱶ *Quotiēt*
The Diuisor.	~~10~~ ʒ + ~~12~~ ⱶ 10 ʒ + 12 ⱶ	
	60 X + 72 ʒ	

The Numbers thus digested, I search how oft 10 in 60, I finde 6 which I put in the Quotiēt adding this Character Seconds, bycauses Thirdes frō Fiftes leaue Seconds, that Quotiēt multiplied in my whole Diuisor, maketh 60 X + 72 ʒ which Deducted frō the Diuisible, leaueth 0. I remoue my Diuisor to ẏ next, searching againe how oft 10 in 80 I finde 8, the which I put in the Quotient with his Character ⱶ bycause 3 frō 4 leaues 1. And for as much as the Characters are vnlike I set down — multiply therfore — 8 ⱶ in that Diuisor, there resulteth 80 ꝣ — 96 ɥ the which Deducted from the Diuisible leaueth nothing, I cōclude therfore 6 ɥ — 8 ⱶ my Quotiēt. The which I may proue whether it be true two wayes, either multiplying the Diuisor with this Quotiēt, so shall you produce the Diuisible number. Or diuiding the same Diuisible by your Quotient, so shall your first Diuisor in the new Quotient be created.

Example.

60 X — 80 ꝣ + 72 ʒ — 96 ɥ	10 ʒ + 12 ⱶ	
6 ɥ — 8 ⱶ 6 ɥ — 8 ⱶ		

I Finde 6 in 60 contained iust 10 times. Put 10 in the Quotient with the Character ʒ bycause 2 out of 5 leaueth 3. Now 10 ʒ multiplied in the Diuisor maketh 60 X — 80 ꝣ which Deducted from the corresponding part of the Diuisible leaueth 0. The Diuisor remoued I search agayne how oft 6 in 72, I finde iust 12, that I put in the Quotient with the Character ⱶ Primes, bycause 2 from 3 leaueth 1, and the Signes being like, I adde + this 12 multi-

plied

plied in 6 ɥ — 8 ɫ createth 72 ℨ — 96 ɥ which deducted from the second part of the Diuisible number, leaueth o. Thus haue ye in the Quotient brought forth your first Diuisor, & therfore are assured, both your works agreeing, that you haue not erred. But if it happen that either the Characters agree not, or the Signes and nūbers so repugne as ye cā not vse this ordinarie course of Diuision, thē place them as a Fraction, drawing a line, and setting the Diuisible aboue, and the Diuisor vnderneath. These kinde of Cossicall Fractions haue their peculiar Rules, which shall hereafter ensue briefly, and so to Æquation.

CHAP. VI.

Of Fractions Cossicall.

THe selfe same Rules that were taught for Addition, Subtraction, Multiplication, and Diuision of common Fractions, serue also in these Cossicall Fragments, as by these Examples ensuing shall appeare.

Examples of Addition.

The numbers Added.

36 ℨ + 16 ϗ / 12 X | 9 ɫ + 4 ɥ / 3 ℨ *Added to* 6 ɥ — 3 ɫ / 4 ɥ | 18 X — 9 ϗ / 12 X

The Product. 18 X + 36 ℨ + 7 ϗ / 12 X

MUltiply the Denominators, so haue ye 12 X the cōmō Denominator, then crosse Multiplication of 4 ɥ in 9 ɫ + 4 ɥ makes 36 ℨ + 16 ϗ, & 3 ℨ in 6 ɥ — 3 ɫ makes 18 X — 9 ϗ. These added make this Fraction.

18 X + 36 ℨ + 7 ϗ / 12 X

Examples of Subtraction.

The Fraction subduced

16 ℨ + 24 ℨℨ | 4 ℟ + 6 ℞ *Deducted* 3 ℞ — 2 ℟ | 24 X — 16 ℨℨ
32 X | 8 ℨ *from* 4 ℞ | 32 X

24 X — 16 ℨ — 40 ℨℨ *The Remaine.*
32 X

Examples of Multiplication.

6 ℞ + 3 ℨ *Multiplied by* 10 ℨℨ *Produceth* 60 ℨℨ + 30 ℨ
4 | 8 ℟ | 32 ℟

5 ℞ + 32 ℨ *Mul.* 8 ℨ + 10 ℞ *Produ-* 256 ℨℨ + 360 X + 50 ℨℨ
7 ℨ — 20 ℟ *by* 6 ℟ — 28 ℨ *ceth.* 602 ℨℨ — 196 ℨℨ — 120 ℞

Examples of Diuision.

By 4 ℟ *I wold diuide* 16 X + 15 ℞ *ariseth of that* + 8 X + 45 ℞
3 | 9 ℞ *Diuision* | 36 ℨ

The Diui. 10 ℞ *The num. to* 12 ℞ + 4 *The Quo-* 192 ℞ + 64
16 *be deuided.* 12 ℟ *tient.* 120 ℨ

These Examples are wrought euen in like sort & forme as the Fractions of Abstract Numbers, and their multiplicatiõ in respect of Signes and Characters, nothing differeth from that which already hath bene declared in Integers Cossicall.

FOr examinatiõ of all these kinds as well Integers *Cossical* as *Fragmẽts*, this one Rule sufficeth. Admit any number what ye list for a *Roote* or *Prime*, & thereby set down in Abstract numbers the value of euery other *Cossicall* nũber.

Then

Then with those *Abstract* nūbers adde, *Subtract*, multiplie or diuide, and confer the *Productes* with *Cossical* numbers produced by these foretaught *Operations*: If they agree, it sheweth a veritie. If ye finde repugnance, Repetition in either kind discouereth where the error resteth.

Of Reduction. Chap. 7.

Reduction is two wayes vnderstood, either to Reduce a Fraction to his least Denomination, or else to Reduce two Fractions of diuerse Denominations to one & the same Denomination.

The first is performed by diuiding both Numerator and Denominator by the greatest common Diuisor that may be found.

The latter by crosse Multiplication of the Numerator of one in the Denominator of the other, wherby resulteth ỹ new Numerators, & then multiplying the Denominators one in another, is Produced the common Denominator.

Example.

I desire to reduce $\frac{16\,\mathfrak{c}}{12\,℞}$ to a lesser Denomination, I diuide therefore both Numerator and Denominator by $4\,℞$ so ariseth this Fraction $\frac{4\,\mathfrak{z}}{3}$ Equal to the former Fraction. Or if it happen that diuerse Fractions be coupled togither by + or − diuide euerie member seuerally by the greatest common Diuisor. Thus: $\frac{16\,\mathfrak{c}}{12\,℞} + \frac{15\,\mathfrak{z}}{12\,℞}$ is Reduced to $\frac{4\,\mathfrak{z}}{3} + \frac{5}{4\,\mathfrak{z}}$ diuiding the first number of the Fraction by $4\,℞$ and the latter part by $3\,℞$. And thus of all such like.

The latter kind of Reduction by crosse Multiplication is already shewed in the Examples of Addition: but for more plainesse I will giue one other Example.

$\frac{12\,\text{ȝ}}{3\,\text{ff}}$ *and* $\frac{8\,\text{%}}{4\,\text{ſſ}}$ *Reduced to one Denomination stand thus.* $\frac{48\,\text{X}}{12\,\text{ȝ}}$ $\frac{24\,\text{X}}{12\,\text{ȝ}}$

HEre 3 ff multiplyed in 4 ſſ make 12 ȝ y^e cōmon Denominator, & 12 ȝ multiplyed crosse in 4 ſſ maketh 48 X. Likewise 8 % in 3 ff augmented, createth 24 X so are the two proponed Fractions reduced $\frac{48\,\text{X}}{12\,\text{ȝ}}$ being as much as $\frac{12\,\text{ȝ}}{3\,\text{ff}}$ and $\frac{24\,\text{X}}{12\,\text{ȝ}}$ no lesse then $\frac{8\,\text{%}}{4\,\text{ſſ}}$ and betwéene themselues of equal Denomination. This Rule is vniuersall for all Fractions, of how many Sections or members so euer they consist. Now to Æquations.

Of Æquations.

ÆQuation is nothing else but a certaine conference of two numbers being in value equal, and yet in multitude and Denomination different.

As we may say 1 Pound is equal to 20 shillings, or 3 Pounds equall to 12 Crownes, or 4 ſſ is equal to 8 ff or 3 ȝ is equal to 6 ſſ. In all these their value agrée, albeit the Numbers and Denominators or Characters be diuerse. Of these Æquations I will speake the more particularly, for that all the Operations of *Algebra* tend to this finall end, to frame an Æquation, and then therby to search the value of the Roote, or Prime, whereby the most difficult questions that may arise or be proponed, are with farre more facilitie to be resolued, than by any other Rules whatsoeuer.

Of

Of Reduction of Æquations.

THe Reduction which is sought in Æquation, is to bring one part of the Æquation, to one simple Cossicall number viz. 1 ɸ, 1 ʒ, 1 ꝶ, or so forth, the which is done two wayes.

First, by transposing or remouing of numbers from one part, to the other.

Then by reducing those numbers so transposed, to their least Denomination, or if they be Fractions, to an Integer.

Of transporting of numbers in Æquations.

THis is a Rule generall, euerie number transposed changeth his signe, as if I say 10 ɸ + 4 are equal to 3 ꝶ, I may trãsfer 4 to 3 ꝶ, & say 10 ɸ are equal to 3 ꝶ — 4. Likewise, as 2 ʒ + 10 ɸ are equall to 3 ꝶ — 6, so is 2 ʒ equall vnto 3 ꝶ — 6 — 10 ɸ, and this for transporting of Signes sufficeth, whereby alway you may reduce one side of the Æquation, to one particular Cossicall Number.

This kind of Reduction by transportation, must be so ordered, that you single by himself the greatest Character, so as the same may stand solitarie on the one side, and the lesser Characters frame the contrary part of ỹ Æquation.

Of Reduction of the parts of an Æquation, to their least *Denomination.*

ONe part of the Æquation being reduced by transportation of numbers, to one simple Cossicall Character. To reduce the Æquation to a lesser Denomination, you shal diuide either part by some common Diuisor, the greatest you can finde.

As if 3 ꝶ be equall to 12 ʒ — 9 ɸ diuiding by 3 ɸ I find 1 ʒ equall to 4 ɸ — 3.

Sometime it shalbe requisite to take awsy some number from either part of the Æquation, as if I haue 6⊬ Equall to 12⊬--24, deducting from either part of the Æquation 6⊬, there resteth ∅ Equall to 6⊬--24, and therefore of necessitie 6⊬ is equall to 24, for this Rule is generall. That if you bring an Æquation (by such Deduction) to a ∅ on the one part, there must be some member in the other connexed with the Signe Minus, the which is alwayes Equall to all the rest of that part of the Æquation.

Sometimes Reduction is made by adding togither all such parcels, as on the one side of the Æquation haue equall Characters, as if 1 χ be Equal to 3 ʒ + 16 ⊬ — 1 ɥ — 10 ⊬. Héere by adding + 16 ⊬ to — 10 ⊬, there resulteth + 6 ⊬, so that I say 1 χ is equall to 3 ʒ + 6 ⊬ — 1 ɥ, and ỳ same diuided by 1 ⊬ maketh 1 ȣ Equal to 3 ɥ + 6 — 1 ⊬.

Reduction of Fragments which shall happen in Æquations to Integers.

ANother kinde of Reduction there is of Fragments to whole numbers, which commeth in vse when an Æquation is found betwéene Fractions on the one or both parts, as if $\frac{4 ɥ + 2 ʒ}{2 ⊬}$ be Equall vnto $\frac{3 ʒ — 2 ɥ}{1 ⊬}$, by crosse multiplication of the Denominator of the one in the Numerator of the other, I finde these two numbers produced 4 ʒ + 2 ȣ, and 6 ȣ — 4 ʒ. Betwéene these, the like Æquation remaineth, & the same first reduced by trāsporting of Signes, maketh 4 ʒ Equall to 6 ȣ — 2 ȣ — 4 ʒ. Then by Addition of 6 ȣ to — 2 ȣ, there resulteth 4 ȣ — 4 ʒ, equall to 4 ʒ. Again, diuiding either part of ỳ Æquation by 4 ɥ, there resulteth 1 ⊬ Equall to 1 ɥ — 1 ⊬. And last of all, deducting 1 ⊬ from both parts of the Æquation, I find ∅ equal to 1 ɥ — 2 ⊬, and therfore of necessity, as was

declared

declared before 1 ℞, equall to 2 ℞. Thus of that intricate Fraction, you ſée how we haue by the former Rules produced this plaine and facile Æquation. Theſe Examples well laboured, will make all Æquations familiar.

The demonſtration of all theſe rules, hang on the third common ſentence, and the fiftenth Propoſition of the fifth Booke of the Elements of Geometrie.

CHAP. VIII.
The Rule of Coſt. or Algebra.

THis Rule is of ſuch perfection, that it performeth not onely whatſoeuer may be done by the rule of *Proportion*, the rule of falſe *Poſitions*, the rule of *Allegation*, the rules of *Archindus*, ſix quãtities of *Cataym*, or of any or all other *Rules* that euer haue bene inuented, but alſo with ſuch facilitie and ſenſible *Method* proceedeth in all his operations, that it may wel be accounted the *Prince* and *Gouerneſſe* of al other. Leauing therfore to waſt words in ſuch by branches, I will bring thee to the *Fountaine* head, whence all other *Rules*, as particular Chanels are deriued. The *Rule* enſueth.

The Rule.

FOr the number ſought, ſet downe 1 ℞, then proceede in your Arithmeticall workings, according to the forme and nature of the queſtion, till you come to ſome Æquation, the which being reduced, as is before taught, you ſhall by the number of that part of the Æquation, which conſiſteth of one ſole Coſſical, diuide the other part of the Æquation onely, if your ſole Character be ℞. Or from the ſame extract ſuch a Roote, as the Character of your ſolitarie ſide demonſtrateth. The firſt Quotient or Roote thereof, ſhall be the vnknowen or deſired Number.

Example.

This Rule cannot be better explaned, than by example. Admit therefore I am demaunded what number that should be, whose third and fourth parts ioyned, maketh 14. According to the rule I say, it is 1 ꝝ viz. one Prime, or one Roote, as commonly Algebricians tearme it. Now the whole cunning resteth in discouering the value of this Prime or Roote.

I reason therefore thus, if the number I seeke be one Prime, then is $\frac{1}{3}$ and $\frac{1}{4}$ of a Prime added togither 14, but $\frac{1}{3}$ ꝝ, & $\frac{1}{4}$ ꝝ added, maketh $\frac{7}{12}$ ꝝ therfore is $\frac{7}{12}$ ꝝ equall to 14. Behold an Æquation, the which because it is already simple and neede no Reduction, according to this Rule with $\frac{7}{12}$ being the number of my solitary Character, I diuide 14, the other part of my Æquation. Of this Diuision, resulteth 24, I conclude therefore 24 to be the number sought, whose third and fourth part added, should make 14. In this Æquation, because the solitarie Character is ꝝ, the quotient of the Diuision discouereth the number sought, but if the solitarie Character had bene ʒ, then should you haue extracted from your Quotient 14 his Quadrate roote, if ꝫ the Cubike Roote.

Another Question.

THere is a band of Souldiers armed with three sortes of weapon, Pikes, Halberdes, and Shot. The Halberds and shot put togither, are double so many as the Pikes, and the Pikes and Shot togither are eight times so manie as the Halberds. And the shot by themselues alone, are in number more thā both the other weapons by 55. I demand the number of Souldiers in that band, and the number of euerie sort of weapon.

For the number of the Halberds, I put 1 ꝝ, then must both the other weapons togither be 8 ꝝ, being by Supposition 8 times so many, and the Pikemen 3 ꝝ, for so doth the

residue

reſidue (being 5 ℞ for the ſhot, added to 1 ℞ for the Halberds) become double to 3 ℞ the Halberds by Suppoſition, Now ſeeing in my queſtion it is ſaid, that the ſhot are more than both the reſt by 55, I adde 1 ℞ my Halberds, to 3 ℞ my Pikes, there ariſeth 4 ℞, which deducted from 5 ℞, the ſhot leaueth 1 ℞ equall to 55. Thus haue I an Æquation. Now diuiding 55 by 1, the number of the ſolitarie Character, there ariſeth in the Quotient againe 55. I conclude therefore 1 ℞ to be 55, and ſo 55 Halberdiers in that band. Then the Pikes being 3 ℞, muſt be 155, and the ſhot being 5 ℞ are 275. All theſe added togither, make 495, thus finde I fiue hundred Souldiers ſaue fiue to be in the whole band. But becauſe ſuch Æquations many times ſhall happen, as the ſolitarie Signe ſhal be a ℞, I will in the next Chapter ſhew how to extract the Quadrate Roote of ſundrie ſorts of Æquations.

CHAP. IX.

How to extract the ſecond Radix or Quadrate roote of any *Coſſicall* numbers, ariſing in any Æquations.

FOraſmuch as alreadie in Reduction, I haue taught ſo to order the Æquation, that ye Characters of the ſecond part of the Æquation ſhal alway be leſſe than the ſingled Character. This is moſt certaine, that the extraction of the ſecond Roote, or ſquare Radix of Coſſicall Numbers, is neuer in any Arithmetical operation neceſſarie, ſaue onely of theſe varieties following.

To finde the Quadrate or ſecond Radix of
- A Number alone.
- ℞ or Rootes onely.
- ℞ + Numbers.
- Numbers — ℞
- ℞ — Numbers.

These are the only fiue varieties, wherin the extraction of Rootes Quadrate **in theſe** Coſſicall Æquations **are required, and for euery of theſe briefe Rules ſhall enſue.**

Rules.

1 For the firſt, I referre you onely to the firſt Booke of my Fathers, where you are taught out of any abſtract or ſimple number or Fraction, to extract his Square *Radix*.

2 In the ſecond, the number it ſelf that is with the rootes coupled, is the number or *Radix* deſired.

3 In the third, you ſhall take the moitie of the number of ℞ or Rootes.

This moytie ſquare, adde the ſame to the Number Abſtract in your Æquation.

To the Quadrate *Radix* of that Product, adioyne the moytie firſt vſed.

The Number reſulting of this laſt Addition, is your deſired *Radix*.

4 In the fourth, ſet downe (as before) the moitie of the ℞ number that N. Square. That Square adde to the number Abſtract, and from the Roote Quadrate of the Product, deduct the moytie firſt ſet downe. This Remaine is the deſired Roote.

5 In the fifth kind there is alway two Rootes, vnleſſe the moytie of the ℞ ſquared, be equal to the number Abſtract, for then is the moytie of the ℞ number the *Radix* ſought. But howſoeuer that fall out, you ſhal as before reſerue the moytie of the Primes, firſt ſquaring the ſame, and from that Square, deducting the Abſtract number. The Roote Quadrate of the Remainder, added to your reſerued moytie, maketh the greater *Radix*, the ſame quadrate roote deducted from the moytie reſerued, leaueth the leſſer *Radix*.

The firſt and ſecond of theſe Rules, **being of themſelues manifeſt, néede no farther explanation. Of the other thrée, particular examples ſhall enſue.**

Example

Example of the third Rule.

1 ȝ ⋲ 6 ꝛ + 27.

Admit this Æquation 1 ȝ ⋲ 6 ꝛ + 27. The Moytie of 6 is 3, that Squared, is 9, which added to 27, maketh 36, the Roote Square of that is 6, whereto adioyning 3, the Moitie first vsed, I make 9, the *Radix* of that Æquation.

Example of the fourth Rule.

1 ȝ ⋲ 80 — 2 ꝛ

Here you are to Extract the Square Roote of 80 lesse 2 Primes or Rootes. The Moytie of 2 is 1, that Squared maketh 1, this added to the abstract number, maketh 81 his Roote square is 9, from that I deduct 1 my first Moytie, so resteth 8, the *Radix* of that Æquation.

Example of the fifth Rule.

1 ȝ ⋲ 14 ꝛ — 33.

The Moytie of the number of Primes is 7, that squared, maketh 49, from this I deduct 33, the abstract number, resteth 16, whose Roote 4 added to 7, the Moytie Fundamentall, maketh 11, the greater Roote, deduct the same 4 from 7, resteth 3 the lesser *Radix*. The truth whereof is thus apparant, square 11 ariseth 121, the Square which should be equall to 14 Rootes lesse 33, 14 times 11 maketh 154 the nũber of the Rootes, from this deduct 33, the abstract number resteth 121 your Square. In like sort, ẏ lesser Roote 3 squared, maketh 9. Now 14 of these Rootes are 42, from which deduct 33 resteth 9 the Square. And hereby it is manifest, that both the one and the other are true Rootes of this Æquation, and mo than these it is impossible to finde.

AMASIAS.

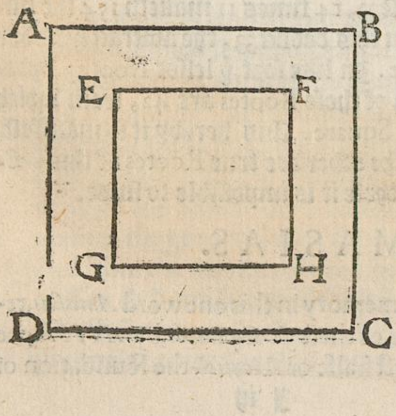
STyfelius for aide of memory in this one word *Amasias*, representeth the rules of these 3 *Æquatiõs*, by *A* or *Ha*, we may remember the first halfe or *Moytie*, the foundation of all the operatiõs,& therfore may be termed the *Moytie Fũdamẽtall*, *M* signifieth *Multiplicatiõ* of that *Moytie* in it self, which I name *Squaring*. *A* & *S* *Addition* and *Subtractiõ*, *viz*, in the two former *Æquations*, to adde the *Square* of the *Moytie* to the abstract nũber, in the last to *Subtract* the one frõ the other, *I*, signifieth the *Inuentiõ* of the Square root of the *Product* or *Remaine*, thẽ *A* & *S* again admonisheth *Additiõ* or *Subtractiõ* of this *root* to be made to or frõ the *moity Fũdamentall*, in forme as the *Signe* of that *Moytie* declareth.

CHAP. X.

Questions concerning the office of the Sergeant Maior.

The first Question.

THere is deliuered to the Sergeant Maior 60 Ensignes, in euery Ensigne 160 Pikes, and short weapon. The Generals pleasure is, that he shall put them into one maine Squadron, and to arme it rounde with seuen ranckes of Pikes, I demaund how many Pikes, how many Halberds, he shall vse to make the greatest Squadron, and how many ranckes shall be in that Battaile.



For resolutiõ of this demaund, First I set down in Portrait the forme of the Battaile here represẽted, by ABCD. The Squadron of shorte weapõ EFGH. The residue representing the 7. Ranks of ẏ Armed Pikes. Seeing therefore I haue 60 Ensignes, & in euery Ensigne 160 of Pikes and short weapō. I multiply 160 by 60, resulteth 9600 the number of men deliuered to be Embattelled. Now since I am demaunded the number of either weapon, and also the number of ranckes represented by the line A D or A B, I say that is 1 ℞ according to the Rule of Cosse. This multiplied in it selfe, maketh 1 𝔷 the Square A B C D, the which is Equall to 9600 the number of men to be imbattelled. Thus am I come to this Æquation 1 𝔷. ∋€ to 9600. The greatest Roote of Integers is 97, the line A D or A B *Viz.* the number of Ranckes.

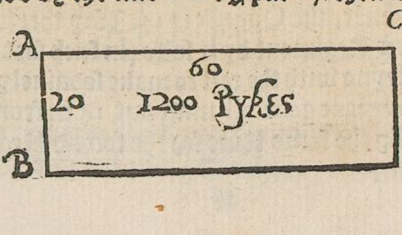
Now considering E F the side of the Squadron of short weapon is 14 lesse: (being armed round which 7 Ranckes of Pikes.) I Deduct 14 out of 97, remaineth 83 the side E F, whose Square is 6889 the number of the Short weapon, and that Deducted from 9409, the Square of 97, leaueth 2520 the number of Pikes. Thus I finde a Resolution of the demaundes as followeth.

The Nūber of Pikes. } 2520 *The nūber of Short weapō.* } 6889 *Rākes in all.* } 97 *Rankes of Short weapō.* } 83

The second Question.

THere are 3. Regiments and in the first Regiment there are but 280. Pikes. In the second 320. In the third Regimēt 600. Pikes. The Generall commaundes them to be put in Battaile of Frount triple the Flanck. I demaund how many Pikes must be in a Ranck, and how many Ranckes, and also in what sort the Sergeant Maior may most sodainly & readily imbattell them.

To resolue this Question for the number of Ranckes represented by the line A B, I put 1 ℞ then must A C, the



front being triple to the flancke, be 3 ℞. Theſe multiplied together make 3 ʒ the whole content of B C the battaile. And ſo ſtands the Æquation 3 ʒ Æ 1200. Pikes, for ſo many pikes are in the 3. Regiments. And by reduction it is 1 ʒ Æ to 400. pikes. The roote quadrate therfore being 20 is the value of one ℞. I auerre therfore the flank A B 20. and the front being triple the flanke 60. and ſo conclude you muſt haue 60. pikes in ranck, and 20 banckes to frame your commaunded battaile. But to reſolue the ſecond part of this queſtion concerning the moſt ſpéedy & compendious way to put men in battaile. There are onely two courſes: ÿ one by reſoluing ÿ groſſe body into Maniples or Herſes of 5. or 6. or ſeuen in a ranck, that may containe as many ranckes as the flancke of the battaile. The other to haue thoſe your Maniples of as many rancks, as your front cõtaineth ſouldiers.

The firſt requireth a great number of Maniples, but theſe marching vp one cloſe to another are preſently in battaile without any moreado.

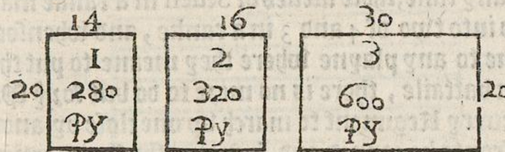
The ſecond hauing fewer Maniples, is ſooner brought into one body: But bycauſe in ioyning their front is made their flancke, they muſt be taught the quarter turne, and ſo are ſodainly in battaile with their front, as it ought to be: And this is in déede in theſe long battailes the more compendious and ready way. As by theſe Examples and figures enſuing ſhall more euidently appeare.

The first Order.

Hauing by the Rule of Algebra foũd that to frame this battaile of triple proportion, the flanck or nũber of rancks muſt be 20 pikes diuiding therewith 280. the number of the firſt Regimẽt, the Quotiẽt is 14. I ſay therfore to embattaile ÿ firſt Regiment by it ſelfe (in ſuch ſort as it may be ready to ioyne with the reſt to make ſodainely that battaile) the Sergeant generall muſt put 14 in front and 20 in flanke. By the ſame Rule for the ſecond Regiment ye

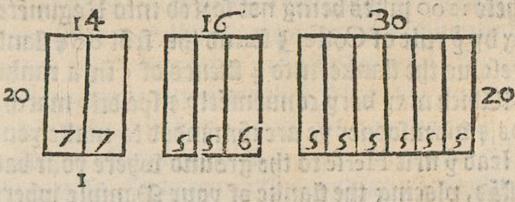
muſt

must put 16 in a ranke and 20 rankes: and the third shall haue 30 in front and 20 rankes: for the number of ranks in all must be 20, cōsidering ye found by your *Æquation* the flanke of your mayne battaile to be 20, and your front 60. as by these figures ensuing more manifestly shall appeare, where you see the 3. fronts 14, 16 and 30. make 60, and so these 3. battalions of these three seueral Regiments



marching in this maner, may sodainly conioyne and make the massie battaile of triple proportion *viz.* 20 in flank and 60 in front that by the Generall was commaunded.

But bycause 30 in a front is too great to march with any speede although it were in Champion, and that by reason of straights or abrupt passages, ye should be many times forced to breake these bodies in your march. It is requisite to reduce euery of these bodies into Maniples of fiue, or six or 7. in a ranke: The first therefore hauing 14 in front I



would reduce into two Herses or sleeues of seuē in a rank: The second Battallione into 3 Maniples, the two first of

5 in a ranke and the third of 6. and the pikes of the third Regiment being 30 in front, ye may resolue either into 5 sléeues of 6. in a ranke, or into 6. sléeues of fiue in a ranke at your pleasure, as by the figures preceding more euidently you may conceiue.

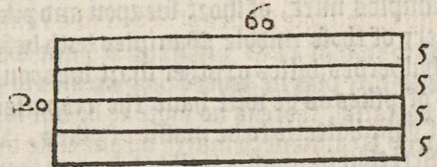
Thus if euery Regiment be drawne out in Maniples of 5.6.or 7 the most in a ranke, they may easily and spéedily march away in all grounds, or if necessitie should enforce at any time, those sléeues of Seuen in a ranke may be resolued into two of 4 and 3 in a ranke, and whensoeuer they come to any playne where they meane to put themselues in battaile, there is no more to do but for ye Maniples of euery Regiment to march vp one close by another till the fronts be ioyned and so make the thrée particular Battallions, and then those battallions either by a like direct march, or if néede be by a counter march well directed by the Corporalles, to vnite themselues, which by this good order and direction first giuen is done sodainly and without any confusion in the tenth part of the time that otherwise it would be. But if your pikes should not be sorted in Regiments, but deliuered you in grosse by the Generall with commaundemēt spéedely to embattaile them, then would I aduise the Sergeant Generall to take the other course as the more spéedy and compendious.

The second Order.

These 1200 pikes being not sorted into Regimēts, cōsidering by ye rule of Cosse I found the frōt 60 & flank 20 I will resolue the flanke into 4 sléeues of 5 in a ranke & 60 rāks. Which may very conuenientlye & spéedily march in all groūds, & whensoeuer ye are cōmanded to make your battaile, lead ye first Herse to the ground where your battaile shall stād, placing the flanke of your Maniple where you meane the Frōt of your battaile shal be, & then cause your Corporals to bring in the other Maniples marching close

vp

vp by yours, till their frõts ioyne matching their rankes equally, the which considering there is but 4 Maniples is done sodainly, & thẽ haue ye no more to do but the quarter turne to make their flanke their front, & your battaile is perfitly formed and ready to ioyne with their enemy as by this figure ensuing will the better appeare.



I hold it néedelesse in these things of facilitie to spend more wordes albeit I know some Italians haue framed bookes of Tables onely for making of battailes of fronts double to their flankes, so that by this Rule of Cosse if I should make like Tables for treble, quadriple, sesquialter sesquiterce, and for all such varietie of proportions of battailes as in ẏ field shal sometimes be vsed, it would amoũt to a volume as big as fiue Bibles, and yet by this one easie and facile rule dispatched with more facilitie, & also more readily, then it is almost possible to turne the booke, and by Suruey of the Tables to finde the number: whereas those Tables, (although they were as great as twenty Bibles) were not able to containe meanes sufficient for all varietie of formes and numbers that this rule extendeth vnto. And to speake plainly a truth, he deserues not to beare the office of a Sergeant General, that is not able of himselfe without aide of any mans Tables (which may happily be miscast or mysprinted,) that is not able of himself I say to correct & reforme such Tables, or without such directions of other men to discharge his office: which in great numbers is vtterly impossible without art to be performed, but so slowly & vnorderly as shall make the officer ridiculous euen to the common souldier.

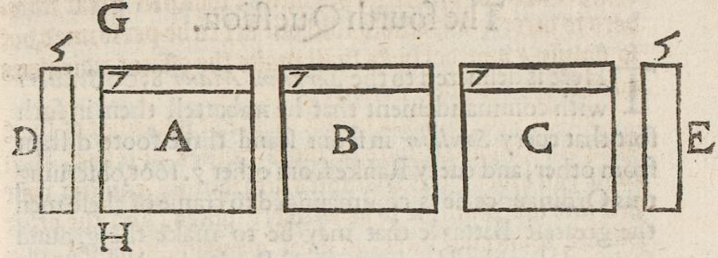
But if your battaile be not massie of pikes but of short weapons empalled with certaine rankes of pikes, then must you order your Maniples accordingly, causing your first and last Maniple to be entierly pikes, and the other middle Maniples mixt, of short weapon and pikes, placing in euery of those middle Maniples both before & behind your halberdes, billes or other short weapons so many rankes of pikes as ye will haue the empallement of the sides of your battaile to be massie of pikes. But this common discretion in practise will teach, and therfore superfluous in so playne a matter to spend more wordes.

The third Question.

THe high *Marshall* commaundeth that the Army shall be diuided into 3 like square Battailes, euery Battaile to be Armed in the front with seuen Rankes of Pikes, and that these battailes making one front vpon the Enemie to be empaled on either side with a sleeue of Pikes of fiue in a rank. This order being prescribed, he deliuereth the *Sergeant Maior*, 18000 *Souldiours* pikes and short weapon, cōmaunding that the *Squadrons* be made as great as possible may be of those men. It is demaunded how many in euery ranke of the *Battallions*, and in what sort the *Sergeant Maior* shall shift his weapons: how many pikes for the sleeues, and how many of short weapons and pikes in euery Battallion seuerally.

To resolue this Question, first I put the Marshals order in figure, A B C representing the 3 Battailes armed in the front with seuen rankes of Pikes: D and E the Sleeues fiue in a Ranke.

Now



Now because I know not G H the side of the Battaile or length of the sleeues, I suppose that 1 ℞, the which increased by 5 maketh 5 ℞ for one sleeue, that doubled maketh 10 ℞ for the 2 sleeues, then square 1 ℞ so haue I 1 ʒ the quantitie of one battaile, and so consequently 3 ʒ for the 3 Battailes. Which adioyned to 10 ℞ the 2 sleeues maketh 3 ʒ + 10 ℞ for Battailes and sleeues. But that should be 18000, for so many men were deliuered to make the Battails. Behold therefore the Æquation 3 ʒ + 10 ℞ ∋€ 18000, which reduced maketh 1 ʒ + $\frac{10}{3}$ ℞ ∋€ 6000, & so consequently 1 ʒ ∋€ 6000 — $\frac{10}{3}$ ℞. The value of this Roote by the fourth Rule is found to be 75 and certaine Fractions which alwayes in these Militarie questions may be omitted. I conclude therefore euerie battaile must haue 75 in a ranke, and that multiplied by 5 maketh 375 the number of Pikes in either sleeue. And because the Battailes are armed in the front with 7 rankes of Pikes, I multiply 7 in 75, so haue I 525, ẏ number of Pikes in euery Squadron, and that deducted from 5625 the Square of 75, resteth 5100 the number of the short weapon in euery Squadron. Thus standeth euerie demand resolued as followeth.

Pykes in either Sleeue. } 375. *Pikes in euery Squadrō.* } 525. *Short weapō in euery Squadrō* } 5625.

The number of Rankes in euerie Squadron. } 75.

The fourth Question.

THere is deliuered to the *Sargeant Maior* 8500 *Soldiers* with commandement that he imbattell them in ſuch ſort that euery *Souldier* in front ſtand three foote diſtant from other, and euery Ranke fiom other 7. foot, obſeruing this Ordinaunce he is commaunded to frame of theſe men the greateſt Battayle that may be to make the ground ſquare. I demaund how many in a Ranke, and how many Rankes in that battayle.

For the number of Rankes I ſuppoſe 1 ℞ then muſt the number of Souldiers in a Rancke be $\frac{7}{3}$ ℞ theſe multiplyed togither make $\frac{7}{3}$ ʓ Equal to 8500 the whole nūber, which Æquation reduced ſtādeth thus, 1 ʓ ∋€ 3643 $\frac{1}{3}$ and ſo by the firſt Rule of Æquations. I finde the Roote 60 with a Fraction which heere neglected I affirme 60 Rankes in that Battayle, which multiplied by 7 and deuided by 3 maketh 140 the number of Souldiours in euery Ranke.

The number of Rankes. } 60 *The number of Souldiers in euery Ranke.* } 140.

HOw by this Art of *Algebra* or Rule of Coſſe as the *Italians* terme it. The *Sergeant Maior* or *Sergeant generall* may readily imbattel his *Souldiers* in what order ſoeuer it ſhall pleaſe the Generall or high Marſhall to command.

INfinite are the formes of imbattelling, ſome Circular, as appeareth by Iulius Cęſar in his Commentaries when he declareth how his Armie Globo Facto, defended themſelues, ſome Triangular, as in the ſame Commentaries may be ſeene where his Souldiers being enuironed of their enemies Cuneo facto erupêre: Some ranged in forme of Lunula, wherein the Turke eſpecially delighteth, repoſing a certaine ſuperſtitious confidence in the figure. But whoſoeuer

ſoeuer ſhall wel conſider howe impoſſible it is for any of theſe or like formes to March and mayntaine their aray, as Souldiours in our age are trayned, ſhalbe compelled to confeſſe the Squadron and Battels made and framed of Squadrons, onely in theſe our warres able to be vſed. And no doubt if the Turkiſh Army march & mayntaine that forme, it muſt be by deuiding the ſame into many Squadrons, placing them in ſuch Lunular forme, and then euery of thoſe Squadrons by the direction of diſcreete Colonels marching vnited, may mayntayne their aray, and ſtill repreſent that their Lunular Figure. The like I ſuppoſe of the Romanes, whoſe Legions were altogither componed of Square Cohorts. Seing therefore the beſt and ſureſt, or rather the onely kinds of imbattelling as well to March as to Fight, is Square or componed of Squares, it ſhall be onely néedfull for the Sergeant General to ſet downe in Figure the manner of imbattelling by the Generall commanded, as I haue in the former queſtions alreadie ſhewed, and then accordingly to procéede in his Operations, allowing for euerie Squadron 1 ꝶ and for euerie number of Souldiers anſwering to the ſide of a Battalion in impalement or otherwiſe, onely allowing 1 ꝑ ſo procéeding till he come to the Æquation.

Finally, by the Art before taught ſearching out the value or number of the ꝑ and ꝶ, he hath the number of Soldiers in euery Battalion, and the number of Souldiers in euerie ranke, togither with the numbers of Souldiours and Rankes in euerie impalement, wings &c. as by the former queſtions and their operations may euidently appeare. And by this Art the Sergeant General ſhalbe able ſuddainely, how great ſoeuer the multitude be, to change their forme of imbattelling into as many ſundrie orders as ſhall be enioyned, or in reſpect of the Enimy, or the place, ſhalbe found conuenient.

CHAP. XI.

Certaine Queſtions touching the Office of the high Marſhal and Campe Maſter.

ALthough the forme of *Camps* may be altered according to the diuerſitie of *Situations*, in reſpect of riuers, or woods that do adioyne thereto, yet for lodging both *Horſemen* and *Footemen* commodiouſly, readily and without confuſion, there is none better than the *Square*. It behoueth therefore thoſe officers to vnderſtand firſt what quantitie of ground ſufficeth for the lodging and encamping of ſome certaine *Regiment* of *Horſemen* and *Footemen*, which knowne by the *Rules* enſuing, he ſhalbe able to extend the ſame to al numbers, and to know readily vpon the view of any ground what number it is able to receiue both of *Footemen* and Horſe, and accordingly to giue order to inferiour Officers in what ſort they ſhall proceed to diuide their ground for euerie *Regiment*.

The firſt Queſtion.

ADmit I finde by experience that 3000 footemen may commodiouſly be encamped in a plat of ground 300 pace *Square*. I demand how many pace the ground ſhould be ſquare that ſhall receiue 10000. Footemen.

To reſolue this Queſtion, I ſay the number of pace demaunded is 1 ℞ the *Square* therof is 1 ʒ of paces, the which is able to receiue the 10000. Then ſay I thus by the Rule of Proportion 10000 giueth 1 ʒ of paces, what yeldeth 3000, by Multiplication and Diuiſion, I finde the fourth Proportional number $\frac{3}{10}$ ʒ whereby it appeareth, that in the Camp of 3000 there is $\frac{3}{10}$ ʒ of paces, but in that Campe by ſuppoſition there is 90000 paces, for ſo much is

300 multiplied in it ſelfe. Behold therefore your Æquation.

$\frac{3}{10}$ ʒ ∋€ 90000 *Paces.*

The Value of the Roote found by the firſt Rule of Æquations is betwéene 549 and 550: ſo many paces therefore ought the ground to be Square that ſhall receiue 10000 men.

The ſecond Queſtion.

I Finde by experience that 1000 *Horſemen* will demaund as much ground to encampe on as 10000 *Footemen*, I finde alſo by experience that 550 pace Square of ground will ſuffice to receiue either of them in one maine Squadrō *Campe*, but my deſire is to diuide either of them into 3 *Regiments*, and to lodge them ſeuerally in 3 ſquare *Campes*. I demaund how many pace Square euery of thoſe *Campes* muſt be.

For the number ſought I ſet downe 1 ʒ that Squared maketh 1 ʒ of paces, the Superficiall content of one of the 3 Camps. Therfore ſhall the thrée Camps be 3 ʒ of paces, but thoſe thrée were contained in 302500 paces, for ſo much is the content Superficiall of the Campe of 550 pace Square. Behold therefore your Æquation.

The Æquation.

3 ʒ ∋€ 302500 { *which reduced makes* { 1 ʒ ∋€ $100833\frac{1}{3}$ *paces.*

The value of the *Prime* ſearched as before is taught, is 317 pace, and ſo much Square ought euery one of the thrée Foote Campes and Horſe Campes to be, for the conuenient receite of ſuch Regiments.

The third Queſtion.

EXperience teacheth, that to encampe 6000 Footemen and 600 Horſe, in ſuch commodious maner that al the streetes may be of reasonable breadth, the Market place and place of assembly of sufficient Capacity, roome sufficient for the Munition, with a Ring and trench meete to receiue the *Sentinels* and Souldiers for defence, togither with the Carriages to impale the same Campe. To performe these dueties commodiously, it is requisite the Campe should be 32 score pace Square. But there is deliuered by the Generall 30000 footemen, and 3000 Horse: I demaund what scope of ground the Lord high Marshall or Campe Master should appoint to receiue this company lodging them as commodiously as the other.

For the number of score or paces in the side of this new Campe whose quantitie is desired according to the Rule of Cosse, I set downe 1 ℞ and because the Horsemen and Footemen, in either Campe are proportionally sorted, I adioyne the numbers of Horsemen & Footemen togither, which make 33000 and seeing the side of this new Campe is 1 ℞ of paces, the Superficial Capacitie must be 1 ʒ of paces. I say therfore if 33000. Souldiers, require 1 ʒ paces to encampe, what shall 6600 the number of Souldiers in the lesse Campe: There ariseth $\frac{1}{5}$ ʒ paces, but by Supposition that little Campe contained 409600 paces, for so many are in 32 score Square.

Behold therefore the Æquation.

The Æquation.

$\frac{1}{5}$ ʒ ⋺ 409600 *paces* $\left\{\begin{matrix}\textit{This Reduced}\\ \textit{maketh}\end{matrix}\right\}$ 1 ʒ ⋺ 2048000 *paces*

And so the value of the *Prime* by *Extraction* of the Roote, as is taught in the Chapter of Æquations, falleth out betwéene 1431 and 1432 paces. So is the side of the new Campe 71 score 12 paces.

But if the Horsemen and Footemen had not bene Proportionally sorted to the numbers in your Little Campe: then is the readiest and surest way seuerally, by the Art

shewed

shewed in the first and second Question, to cast how much ground either Horse Campe and Foote Campe will require, allowing cōuenient groundplats for Market place, & places of assembly, with Streets to part the said Campes, and also space sufficient for the Ring of the Camp, proportionally, according to the increase of the numbers of souldiers to be encamped. Finally, adding togither the superficiall content of those Campes, places of assembly, and stréetes &c. there resulteth the superficial quantitie of the whole Campe, which reduced into paces, the Roote Quadrate thereof is the number of paces contained in the side of that Campe. And hereof I néede adioyne no example, considering there is nothing else in this Operation but a reiteration of the former workings seuerally for either Campe, the places of assembly, the stréets, &c. transferring by the Rule of Proportion the measures from one Campe proportionally to another.

The fourth Question.

HOw the high Marshall or Campe master whatsoeuer number of Horsemen and Footemen shalbe deliuered vnto him, may redily coniecture what quantity of ground will suffice strongly and commodiously to encampe them, and how much he ought to allow to either Horsecampe or foote Campe seuerally.

To resolue this Question, it is cōuenient to know what quantitie of ground is to be allowed to a Horseman, and how much to a Footeman, & this is only by experience to be learned, and among the experimented souldiers of diuers ages and Nations, I finde diuersitie of Opinions. Monsieur Lange alloweth to euerie Footemā 90 Quadrate foote of earth, and to euery man of Armes 900, & to euerie Estradiote and light Horseman 800. Others following the Romanes, assigne to euerie Footeman a Quadrate plat of ground sixe foote broad, and tenne foote long, and to euery Horseman one with another the lodging of three footmen. The Romaines sought alwayes to encamp strongly, pressing nigh togither, not regarding pleasure or commoditie. But as their Proportion is somwhat too straight for Souldiorie trayned in this our age, whiche can neyther away with the anoyances, ne yet with the hard dyet, which was familiar to the Romans: so iudge I the former Proportion of Monsieur Lange, ouermuche to dilate and enlarge the Camp, & therefore considering the streetes that deuide the Horse and foote Campe, must be large, and also betweene euery Regiment in either Camp, there must be cōuenient wayes and passages, neither by ouermuche delating the Campe to make it weake, ne yet by ouer streightning of the Lodgings and passages, to pester the Souldiorie, I hold it conuenient to allow to euery Footman, 80 square foote of earth, and to euery Horseman 400. and for the places of Assembly, the Market place, the Butcherie, the Victuallers, and place to receiue the Munition & Ordinance, with the Pioners, and other Mechanical artificers as much more as both Campes amount vnto, so will there fall out, after the Ring of the Campe is described of conuenient largenesse, and all streetes and passages in proportion answerable with due place to receiue the Colonels, Captaines, and Officers of euery Bande particularly, that then the Lodgings of a Footeman shall not amount vnto aboue 50 foote, and euery Horseman one with another 250. To resolue therfore this Question, you shal multiply ẏ number of footemen by 80, for the superficiall content of the Footcamp, and the number of Horsemen by 400 for the Horse camp, the Products adde togither, and double the resulting summe: the Roote Quadrate of this last produced number diuided by 3, delinereth in the Quotient howe many Pace square the Campe shall be, which may both strongly and commodiously lodge your proponed Armie. And the

Rootes

Rootes Quadrate of the contentes superficiall of either Camp diuided by thrée, doth also deliuer how many Pace the side of either Camp shall be.

Example.

Admit I haue 30000 Footemen, and 6000 Horse to be encamped, I multiplie 30000 by 80, ariseth 2400000. I augment also 6000 by 400, ariseth likewise 2400000, these ioyned, make 4800000, which doubled, presenteth 9600000, the Quadrate Roote hereof is 3093, which parted by thrée bring..th in the Quotient 1031. So many Pace shall the side of th.. Square Camp be, which shall lodge the thirtie thousand footemen, and sixe thousand Horse. If the Enemie be at hand, I would haue alwayes the Ring and Trench of the Camp within, but if the Enemie be not doubted, they may be made without this proportion of Camp, whereby the stréetes and lodgings may be enlarged.

The more ample handling of this office referred to another Treatise.

I Might here adioyne sundry formes and orders of Encamping, vsed by sundry Nations, and thereupon frame a great number of *Arithmeticall Questions*, but I reserue the more ample handling of this matter, to another Treatise of *Fortification* of Townes and Campes, wherein I will declare how by the *Topographicall Instrument* described in my *Pantometria*, the High Marshall sodainely may set downe the *Ring* of the *Camp*, that shall receiue any Armie that shall by the *Generall* be committed vnto him, assigning to euery Regiment his *Quarter*, and how with like celeritie the *Camp Maister* or *Prouostes* of euery Regiment shall distribute their Bandes, and the *Harbinger* of euerie Bande his seuerall Lodging, so that the Armie shall speedely, strongly, and commodiously be lodged, without confusion or disorder. And for this present Treatise, beyng onely *Arithmeticall*, these fewe Questions may suffice to giue some light to the *Ingenious Souldiour*, to search further how this *Art* may stand him in stead, for exact ready dispatch in his Militarie Actions.

CHAP. XII.

Certaine Questions Arithmeticall, concerning the Office of the Master of the Ordinance.

IF a Falcon of three inches Bullet weigh 500 Pounde, I demaunde how much a Canon of eight inches will weigh, that is able to receiue his proportionall charge to that Falcon.

This Question by the simple Rule of Proportion, can not be answered, for as much as weight is peculiarly apperteining neither to lines nor Superficies, but onely vnto Solide bodies. Seing therfore as it is by Euclid demonstrate, and in my Pantometria I haue already taught, that Spheres, and all other like & vniforme bodies, be in Triple the proportiō of their Diameters, you shall multiply either Diameter or height of the Bullets Cubically, and with those Cubicall numbers, worke according to the Rule of Porportion, as by the Example ensuing more playnely shall appeare.

Example.

The Cube of 3 is 27, the Cube of 8 is 512. Now by the Rule of proportiō, say 27 yeldeth 512, what giueth 500 the weight of the Falcon, the fourth Proportionall number amounteth vnto 9481. The Cannon therefore that shall be able to carrie a Porportionall charge to the Falcon, must haue 9481 pounds of mettall, but bycause commonly those greater

greater sort of péeces are not so massie of mettall as in déede they ought to be, the Gunners haue it for a generall Rule, that in all péeces aboue six inches of Bore, they must abate $\frac{1}{4}$ of their ordinarie charges.

The second Question.

If a Falconet of three inches the Bullet, require three pound of Powder for his charge, I demaunde how much of like Powder will charge a Cannon of eight inches Bullet.

In this as in the former, the Rule of Proportion plainly vsed fayleth, but working with the Numbers resulting by Cubicall Multiplication, the Quotient will shew the desired waight of Powder.

Example.

I multiplie 512 the Cube of 8 by 3 the charge of the Falcon, there ariseth 1536, which I diuide by 27, the Cube of 3 the Diameter of the Falcõs Bullet, so haue I in the Quotient 59 $\frac{4}{27}$: so many pound of Powder is ye due charge of your Cannon, but if by the Rule last taught, you finde that the Cannon hath not his proportionall masse of mettall, you may according to the vsuall Rule of Gunners, abate $\frac{1}{4}$, so will there remaine 42 Pounds, and somewhat more of Powder for your Canons charge.

In this sort by the Charge of any one péece of Ordinance knowne, you may finde out the certaine Charge of all others.

The third Question.

If a Falcon that carrieth poynt blancke 150 pase, at vtmost randon range 1300 pases, I demaunde how farre a Culuering at his vtmost randon will reach, that at poynt blancke, or leuell, rangeth 250 pase.

I cannot here, but note the grosse errour of Girolamo Ruscelli Nouarese, who in his Booke entituled *Militia Moderna*, intreating of great Ordinance, supposeth, that in all sortes of Péeces, the difference of their vtmost Ranges, should be in proportion answerable to the waight of their Bullets, and charges of Pouder, and thereupon deliuereth a Rule by Multiplication and Diuision (the vtmost Range of any one péece knowen) to finde the same in all other.

In the same Chapter also, he publisheth another Errour, supposing that in one, and the selfesame péece of Ordinance discharged with seuerall charges or quantities of one kind of Pouder, that the ranges of the Bullets should alway be Proportionall to the quãtities of their charges, or waight of the Pouder wherewith they are charged.

In both these cases, the diuersitie of the ranges is compounded of sundry Proportions, as in a peculiar Treatise for that purpose I will declare, and can not by the Rule of Proportion onely be discouered. But in this Question, the Rule of Proportion precisely serueth, bycause the Pointblancke and Vtmost ranges in all Péeces whatsoeuer are in déede Proportionall. I multiplie therefore 250, by 1300, there ariseth 325000, which diuided by 150, yeldeth for the fourth proportionall 2166 $\frac{2}{3}$, so many Paces shall the Culuerine reach at his vtmost Randon.

And thus by obseruations vsed in one Péece, by this Art of Proportion, a man may discouer the force of all other.

Other matters in the office of the Master of the Artillerie to be considered.

I might here adioyne many mo *Questions*, touching the waight, quantitie, and number of Pouder, Shot, and sundrie sortes of Ordinance to be vsed at a *Batterie*: how to mount all sortes of Peeces, to strike any marke at *Randon*:

don: the number of Carriages, of Ladles, Rammers, Scourers, Waddes, Tampions, Cartages, Matches, Barrels, or Lastes of Powder &c. Also, the number of Gunners, Assistantes, Pioners, Smithes, Carpenters, and other *Artificers*, to attend on the *Artillerie*, what number of Horses and Oxen to drawe them, the waight of all sortes of Peeces, the charges of them, their Wheeles and Carriages. Of those and many other things to the office of the Master of the *Ordinance* appertaining, as well for the *Field* and *Camp, Towne* or *Forte*, as also for seruice on the Seas, I might propone an infinite number of Questions *Arithmeticall*: but hauing in a peculiar Treatise of *Artillerie* prepared to handle at large all these, and many mo rare *Experiments* of great Ordinance, I leaue further in this place to wade in that Office.

CHAP. XIII.

Questions Arithmeticall, concerning the Office of the Treasurer, and Auditor, the Master of the Victuals, and Captaine of the Pioners.

The first Question.

If 17000 pounde be able to paye 5000 Souldiers for nine weekes, I demaunde how much treasure will suffice to pay 30000 Souldiers for a yeare.

This Question, as it standeth vpon a double consideration of men and time, so is it also to be double wrought by the Rule of Proportion, saying thus, if 5000 souldiers require 17000, what will 30000 for like time require. By Multiplication and Diuision I find the fourth Proportionall 102000 Poundes. Thē agayne I say, if nine Wéeks require 102000 Pounds, what shal 52 Wéekes, for so many Wéekes are there in a yeare. The fourth Proportionall is, 589333 Poundes and a Noble, so much Treasure will pay 30000. Souldiers for a yeare.

The second Question.

Admit there be of a Praye or Bootie taken 300 Pounds sterling to be distributed to a Bande of 150. footemen, wherein there is 20 Souldiers wanting of a Bande cōplet: I demaunde how much the Captaine and euery seuerall Officer and Souldier of the Bande should haue for their part or share ratably made according to true auncient Disciplinе Militare?

This Question, albeit it séeme facile and easie, yet is it not possible for the best Auditor in Englād by Counters to finde out or set downe their shares or parts exactly, as by Arithmeticke it may be done: But for Resolution of this Question, it is first necessarie to know the Princes List or Rate of payes for such a complet Band Journall or Annuall: And for because there are 20 Souldiers in defect, ye shall from the Totall of the Journall charge, abate 20 Souldiers paye and twentie dead payes more to be checked in respect of that defect, the Remaine is the Journall charge or due of that Band: This Remaine ye shall reserue for the Diuisor generall. Then multiplie the Journall pay of the Captaine, Officer or Souldier (whose share ye desire to know) by the 300 Poundes sterling diuisible among the Band, and the product diuided by the former reserued Diuisor generall, discouereth in the quotient and his appendant fraction the true part exactly to that officer or Souldier due, as by the operations ensuing better shall be conceiued.

Example.

First I set down the List of a foote Band of 150 Souldiers signed and established in the gouernement of the Earle of Leicester, as ensueth.

The

The Iournall Rates of a Bande of 150 footemen complete.

	Per diem. s̄	d
The Captaine.	6	0
The Lieutenant.	3	0
The Ensigne.	1	6
Two Sergeants.	2	0
Two Drummers.	2	0
One Surgeon.	1	0
Totall Officers.	15	6

	l	s̄	d
135. Souldiers and 15 dead payes at 8 d. *per diem le pe.*	5	0	0
The summe Totall of the whole Band complet.	5	15	6

This 5 l, 15 s̄, 6 d, resolued into pēce maketh 1386 pence, from which I abate 176 pence for 20 souldiers, and 2 dead payes defect &c. checked, the Remaine being 1210 d, is the true Iournall charge of that band checked by the true disci- pline of Musters, and this 1210 is my Diuisor generall.

Then doe I multiplie for the Captaines portion 6 s̄, reduced into pence by 300, the summe to be distributed, & the product diuided by 1210 the Diuisor generall yeldeth in the quotient 17 $\frac{1030}{1210}$ l, which reduced in coyne maketh 17 l, 17 s̄, 0 $\frac{36}{121}$ of a peny.

Likewise for a priuate souldiers pay I multiplie 8. by 300, and the product being 2400 I diuide by 1210 the Diui- sor generall, so haue I in quotient $\frac{119}{121}$ pounds, which redu- ced is 1 l, 19 s̄, 8 $\frac{4}{121}$ pence.

And by like ſupputatiō I finde the exact portiōs of euery other officer, as here vnder is particularly ſet downe.

The portions or ſhares of	l.	s.	d.
The Captaine.	17	17	0 $\frac{36}{121}$
ye 13 dead payes diuiſible &c.	25	15	8 $\frac{52}{121}$
The Lieutenant.	8	18	6 $\frac{18}{121}$
The Enſigne.	4	9	3 $\frac{9}{121}$
A Sergeant.	2	19	6 $\frac{6}{121}$
A Surgeon.	2	19	6 $\frac{6}{121}$
A Drumme.	2	19	6 $\frac{6}{121}$
A priuat Souldier.	1	19	8 $\frac{4}{121}$

The Corporals & Gentlemen of the Band are to haue like ſhare & portion to the Sergeant of a band: but the ouerplus aboue the priuate ſouldiers ſhare, is to be allowed thē out of the dead payes, and the ouerplus of thoſe dead payes at the Captaines diſcretion, according to the Militare Ordinances eſtabliſhed by the Earle of Leiceſter, and States Generall, as hereafter in the third booke of Militare laws and dueties more at large ſhalbe declared.

The third Queſtion.

If 1200 Quarter of Corne ſuffiſe 4000 Souldiers for 9 Weekes, how much ought to be prouided to ſerue 25000 Souldiours for 40 Weekes.

Double working by the Golden Rule reſolueth this Queſtion euen as the former. I ſay therefore, if 4000 Souldiours require 1200 Quarters of Wheate, what ſhall 25000: working by the Rule of thrée, I finde 7500 Quarters. Againe I if ſay, nine Wéekes demaunde 7500 Quar-

Quarters, how much will 40 wéekes. The fourth Proportionall resulting by Multiplication and Diuision, is $33333\frac{1}{3}$. I conclude therefore that to maintaine an Army of 25000 Souldiers 40. wéekes, the master of the Victuals must prouide $33333\frac{1}{3}$ quarters of Wheate: in like sort may he forecast for all other Prouision.

The third Question.

If 500 Pioners can in tenne houres cast vp 400 rodde of trench, I demaund how many labourers will be able with a like trench in three houres, to intrench a Campe of 2300 rodde compasse.

This Question hath a farther difficultie than the other two last past, because in the second operation the Rule of Proportion must be inuersed, and wrought backward, as by the Example shall more plainely appeare.

Example.

First I say, if 400 rodde of Trench require 500 souldiers, what shall 2300 rodde. Héere according to the plaine Rule of thrée, I multiplie the third by the second, so haue I 1150000, which diuided by the first, delivereth in the Quotient 2825 Souldiers: then must I say for the second operation, if in tenne houres 2825 Souldiers be able to discharge it, how many shall performe the same in thrée houres? Now if you should worke by the Rule of Proportion direct, you should finde a lesser number of souldiers, because thrée houres is lesse than tenne houres, but because reason teacheth me, that the lesser is the time wherein the Trench must be made, the more Labourers I ought to haue, I must inuerse the operation, multiplying 10 the first number by 2825 the second, and diuide by 3 the third number, so haue I in the Quotient $9416\frac{2}{3}$ so manie Pioners must I haue to intrench that Campe in thrée houres.

How manie wayes the Rule of Proportion direct, and inuersed, serueth the tourne in these forenamed Offices.

I Thinke it not necessarie in these matters to propone any more Questions, seing the ingenious Arithmetician by consideration of these may deuise infinite others, as well in the Treasurers Office to supputate the wéekely, monthly, and yearely charge of euerie Band of Horsemen or Footemen, how different so euer their payes be, as also the charge of Ordinance, Powder, Cariages, and all other sortes of Munition, and Instruments méete to follow any Army.

The Master of the Pioners likewise, and Master of the Victuals shall neuer be able without this Art to put readily in execution the commandements of their superiour Magistrates: and the more perfection they haue in this Science, the more spéedily and with lesse staggering shall they be able to discharge their duetie, and shal not néede to rely vpon the direction of any seruant or other hired person, as many do, that being appointed in Office, wher they should direct others, are fain first to hire some one or other to direct themselues. Or else that worse is, following their owne vnskilfull braine, shame themselues & ruinate their Souldiers. To auoid therfore such inconuenience, I would aduise euery Gentleman that will addict himselfe to the wars, to make this Art first familiar vnto him, the which he shall finde not onely in this Campe and field seruices to enable him: but also in matters of Fortification it is so requisite, that it may by no meanes be spared: As in an especial Treatise that I haue begun of those matters, I wil

make

make apparant. And this worke being meere Arithmetical, may not well be interlaced, the same crauing aid also of Instruments and Demonstrations Geometricall.

Touching the Art of Algebra how much requisite for a Souldiour.

This Art consisting of infinite varietie of Æquations in numbers Cossical, & also of numbers Radicall, Rational & Irrationall, would require a seueral Treatise of great quantitie sufficiently to handle the same, but because the subtile Demonstrations of the most curious Cossicall Æquations, & likewise the exquisite Operations in surde numbers, are rather for exercise of Inuẽtion, & to shew perfection of Science, than for any necessitie to be vsed in matters Politike, or Mechanical, I haue here onely briefly deliuered so much therof, as in a Souldier may not be missed: Such a Souldier I say, as shal in al Militare causes be able to see with his owne Eyes, heare with his owne Eares, and discerne with his own Wits, & not resemble such as run wt others Legs, & flie with others Wings: such I meane, as if euery Bird should pull her owne feather, like Esops Daw might dance naked. These are no curious Inuentions, but plain, easie, requisite Rules ẏ here I haue deliuered: The necessary vse of thẽ in my other Treatise of Artillery, Architecture, Nautical, & Militare, shal more plainly appeare, meaning neuerthelesse (God sparing life & Liberty frõ my long troubles) to satisfie also ẏ expectation of such as fly an higher Gate, & wil not stoop at cõmon Pray: hauing in ẏ latter part of my Pãtometria & in diuers parts of my booke entituled Alæ seu Scalæ Mathematicæ, alredy deliuered sundry Inuẽtions & Demõstrations, neuer yet disclosed by any, which if please thẽ in ẏ meane time, they may peruse. And to such as delight in matter seruiceable for ẏ State in causes

Militare (the same being indeede the Art and Profession onely or chiefely conuenient for the Nobilitie, and Gentlemen of this land) I hope this Introduction shall not be vnwelcome: meaning, as I see the same gratefully accepted, hereafter to impart the rest, leauing at this time farther to wade in the large Sea of Algebra and numbers Cossicall.

ARDVA QVAE PVLCHRA.