Outlines of Natural Philosophy being Heads of Lectures Delivered in the University of Edingburgh

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Pneumatics.

PNEUMATICS.

342. The doctrine of Pneumatics, treats of elastic fluids, as accelerating or retarding the motion of bodies; of air, as the vehicle of sound; and, lastly, of air, as the vehicle by which heat and moisture are distributed over the surface of the earth. It is, therefore, naturally divided into three sections.

Sect. I.

AIR AS ACCELERATING OR RETARDING MOTION.

Machines for raising Water.

343. If a bent tube, open at both ends, have the end of the shorter leg immersed in water; and if

if the air in the tube be sucked out, or otherwise extracted, the water will flow through the tube in a continued stream, providing the height from the surface of the water, to the top of the bending, does not exceed 32 or 33 feet.

A bent tube of this kind is called a *syphon*. If the syphon, instead of having the air extracted by suction, be filled with water, and the ends stopped till it is inverted, with the shorter leg immersed in water, the same effect will follow.

It is the greater weight of the water in the longer leg of the tube that determines the fluid to descend on that side, and the pressure of the air continues the supply.

The syphon may be employed to raise water over a height less than 33 feet, if it is to descend below the level of the fountain on the opposite side. It could not, however, be conveniently applied, if the height was near to 33 feet; because the velocity with which the water in the shorter leg is made to ascend, depending on the difference between that height and 33 feet, the length of the column which the air is just able to sustain, might not afford a sufficient supply for the stream descending in the longer leg.

The Syphon is properly a pneumatico-hydraulic machine, the action of water and of air being both necessary to its effect. It is, however, the simplest of all the instruments employed in raising water.

344. The Air-pump is a machine by which the air is extracted from close vessels. A cylindric barrel in which a piston works, communicates by a valve in its bottom, which opens inwards to the receiver or close vessel. The piston is also furnished with a valve, which opens outward; so that when the piston is thrust down, the air in the barrel opens the valve of the piston, and makes its escape. After the piston has been forced down to the bottom, it is drawn up when its valve shuts, and that in the bottom opens, and admits the air from the receiver. This operation is repeated, and the air in the receiver becomes continually rarer.

The air-pump was invented by OTTO GUERICKE, a gentleman of Magdeburg in Germany, about the year 1654, and marks a great era in the history of the physical sciences. It has received great improvements. CAVALLO, vol. II. p. 465, &c. HUTTON'S *Mathematical Dictionary*, article *Air-pump*.

345. When the number of strokes of the piston increases in arithmetical progression, the quantity of air remaining in the receiver, and also its density, form a series of terms decreasing in geometrical progression; so that if R be the capacity of the receiver, B of the barrel, D the density of the air in the receiver, after n strokes

strokes of the piston, the density of the external air being 1; $D = \left(\frac{B}{B+R}\right)^n$.

Hence $\log D = n \log \frac{B}{B+R}$; from which D may be found when n is given, or n when D is given.

It is evident that the exhaustion can never be complete, as no more than a certain proportion of the air that remains can be taken out by the next stroke of the piston. There is even a limit which it cannot exceed in the ordinary air-pump; for the spring of the remaining air, being the power which opens the valves, when it becomes too weak to overcome their stiffness and their weight, the exhaustion necessarily ceases. In air-pumps of the most improved construction, the valves are opened by a mechanical contrivance, so that this limit to the perfection of the instrument is removed.

346. When the end of a cylindric barrel constructed as above, is either immersed in water, or made to communicate with a reservoir of that fluid, on the piston being drawn up, and the air in the barrel rarefied, the water will ascend on account of the pressure of the external air. By repeated strokes of the piston, it may be made to rise so high, as to pass through the valve in the piston,

piston, and a quantity of it is then lifted up every time the piston is raised.

This machine is called the *Sucking Pump*, and the piston is called the *Sucker*.

The valve of the piston, when lowest, must be nearer to the surface of the water in the reservoir than 33 feet, otherwise the water can never rise above it.

347. The piston of the sucking pump when it is raised, sustains the weight of a column of water, of a base equal to the section of the piston, and a height equal to that at which the water in the pump stands above the surface of the reservoir.

This is the measure of that part of the pressure of the external air that is not counteracted by the elasticity of the air under the piston.

To work the pump, a force must, therefore, be employed able to overcome the weight of this column, and of the piston, together with the friction of the piston against the sides of the pump.

348. In a sucking pump, the height of the lower, or fixed valve, above the surface of the water $= a$, the length of the stroke of the piston $= b$, and the height of a column of water in equilibri-

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um with the pressure of the atmosphere $= h$; the height to which the water is raised by the first stroke, is $\frac{a+b+h-\sqrt{(a+b+h)^2-4bh}}{2}$.

Bossut, *Hydrodyn.* vol. I. p. 101.

349. The same notation being retained, and c being put for $a+b$, or the greatest height to which the piston ascends, b must be greater than $\frac{c^2}{4h}$, otherwise the water will not rise above the piston.

The water will cease to rise, when the air in the barrel of the pump, which is always rarer the higher the water is raised, becomes so rare, that when the piston is thrust down, it is not so much condensed as, by its elasticity, to open the valve in the piston, or, which is the same, is not so dense as the external air. This leads to the limit just assigned.

Besides the pump just described, there is one with a solid piston, without any valve, by the raising of which, the water is drawn up through the valve in the bottom of the pump, and on thrusting down the piston, the water is forced through a pipe in the side, having a valve that opens outwards, by which means it can be raised to any height if a sufficient

ficient force be applied. This is called the *Forcing Pump*.

The Lifting Pump is in principle no way different from the Sucking Pump.

350. The discharge of water by pumps, though naturally intermitting, may be rendered continual by the elasticity of the air condensed in what is called an *air-vessel*. This contrivance is represented in fig. 23.

See Desagulier, *Course of Experimental Philosophy*, vol. II. p. 152, 3d edition. Muschenbroeck, § 124, &c. Bossut, *Hydrodyn.* vol. I. § 76., &c.

Steam Engine.

351. Let a cylinder ABCD (fig. 24.), placed vertically, have a piston working in it, the rod of which is fixed to the end of a beam GH, turning on the axis O, and loaded at its other extremity by a weight W, and when the machine is at rest, let the beam incline toward the side where the weight is. Then if a vacuum be any how made in the part of the cylinder under the piston, the whole pressure of the atmosphere will tend to depress the piston, and will raise the weight W, if its momentum

mentum be not greater than that of a column of water 33 feet high, having for its base the section of the cylinder, and acting by the lever GH.

In this way, an air-pump might be made to raise water to any height whatever. But the production of the vacuum by the mechanical means of valves and pistons, from its difficulty and expence, could not be reduced to practice. For this reason, recourse is had to steam, which, from its facility of being generated and destroyed, affords the means of producing a vacuum over a great extent, and of renewing it at pleasure.

Suppose, therefore, the steam of water from a boiler to be introduced through a valve into the cylinder, and to expel the air; and when this is done, the valve to be shut, and another valve opened, by which a jet of cold water is injected into the cylinder, the steam will be quickly condensed; a vacuum will be produced, and the piston will be forced down by the pressure of the atmosphere.

The introduction of the steam at the bottom of the cylinder will again elevate the piston, and the reproduction of the vacuum will cause the depression of it; so that the end of the lever to which W is attached, may be employed to raise any weight, or to pump up any quantity of water within certain limits. This is the Steam Engine in its simplest state.

352. If b be the length of the stroke of the piston, r the radius of the cylinder, and h the height of the column of water which balances the pressure of the atmosphere, $\pi r^2 h$ is the moving force, and its momentum is $\pi r^2 h \times b$. If the radius of the piston of the pump worked by the beam be r', d the depth of the water, and h' the height to which it is raised at one stroke, $\pi r'^2 d h'$ will be the momentum of the resistance: then, in order that the machine may work, $r^2 b h > r'^2 d h'$.

To a complete theory of the steam engine, much more is necessary than the knowledge, that $r^2 b h$ is greater than $r'^2 d h'$; for the rate of working depends on that excess, and must be determined as in the problem, N° 182. The weight of the pump rods must be included, and also the effect of friction.

It is here supposed, that a perfect vacuum is produced in the cylinder by the jet of cold water. This, however, is not the fact, for by the alternations of the heat and cold to which the cylinder is exposed, it can neither acquire the heat necessary to the full elasticity of the steam, nor the cold necessary for its complete condensation. On this account, the effect of the machine falls much under the computation.

353. When the cylinder is full of steam, if a valve be opened, by which the steam is allowed to

escape

escape into another vessel, where a jet of cold water is introduced, the condensation is much more complete, and the heat of the cylinder being preserved, the steam possesses its full elasticity.

This improvement was made by Mr WATT, and completely changed the character of the steam-engine. In the old engines, the power was reduced nearly to half its real value, so that the moving force, instead of amounting to 14 lb. on every square inch of the area of the piston, was reduced to little more than seven. In Mr WATT's engines, the moving force is not less than 12 lbs. on the square inch.

354. A farther improvement has been made on this engine, by injecting the steam into the cylinder, alternately above and below the piston, so that the whole motion is produced by the elasticity of steam, and has no dependence on the weight of the atmosphere.

This improvement is also due to Mr WATT, and could not have been made without the previous contrivance of condensing the steam in a separate vessel. It is particularly accommodated to the production of a rotatory motion by means of a steam engine.

In the double-stroke engine, the piston rods require to be forced down as well as to be drawn up, in the same vertical line. The method by which Mr WATT has accomplished this depends on a geometrical theorem.

355. If the ends of a line given in magnitude, describe circles given in position, and having their convexities turned opposite ways, a point may be found in it, which will describe on each side of the line joining the centres of the circles, an arch of a curve of contrary flexure, not differing sensibly from a straight line.

The point to be found divides the given line in the ratio of the radii of the circles described by its extremities.

A rectilineal vertical motion is also produced by another construction. Two of the adjacent angles of a parallelogram, are made to describe concentric circles, so that the side between them passes through their centre, and one of the remaining angles another circle, having its convexity opposed to that of the two former; then the third angle of the parallelogram describes a line that differs insensibly from a straight line.

PRONY treats of this motion, *Arch. Hydraulique*, tom. II. § 1478.. &c.; but has not given a complete theory of it. He says, on the authority of ADAMS, (*Geometrical and Graphical Essays*) that the contrivance was suggested to Mr WATT by one of the instruments for describing curves, invented by SUARDI. See *Nuovi Instrumenti per la descrezione di diverse Curve*, &c. We have looked into this work with considerable attention, but have not been able to find any thing that has much affinity to the motions just described.

355. In

355. In steam engines of similar construction, the effects are nearly as the quantity of fuel consumed.

In this estimate time is necessarily involved, as, in order to derive the greatest advantage from a given quantity of fuel, the combustion must neither be too quick nor too slow.

It is computed, that an engine of the best construction, will raise 20,000 cubic feet of water to the height of twenty-four feet for every hundred-weight of good pit-coal. An engine with a cylinder of thirty-one inches diameter, and making twelve strokes in a minute, will do the work of forty horses, and will burn 11,000 lb. of the best Staffordshire coal in a day. See *Encyclopædia Britannica*, article *Steam*, p. 769. Prony, *Arch. Hyd.* § 1499. vol. ii.

Motion produced by Gunpowder.

356. When gunpowder is fired, a permanently elastic fluid is generated, which being very dense, and much heated, begins to expand with a force at least 1000 times greater than that of air under the ordinary pressure of the atmosphere.

Robin's *Tracts*, *New Principles of Gunnery*, Prop. vi. Hutton's *Mathematical Dictionary*, art. *Gunpowder*.

In the acceleration of a ball by the fluid thus generated, three circumstances must be attended to.

357. 1. The elasticity is inversely as the space which the fluid occupies, and, therefore, as it forces the ball out of the gun, it continually diminishes.

2. The elasticity would diminish in this ratio, even if the temperature remained the same; but it must diminish in a much greater ratio, both from the dispersion of heat, and the absorption of it by the fluid itself, during its rarefaction.

3. The air propels the ball, by following it, and acts with a force that is, *cæteris paribus*, proportional to the excess of its velocity above the velocity of the ball. The greater the velocity that the ball has acquired, the less, therefore, is its momentary acceleration.

The effect of the elastic fluid must, for these reasons, decrease much faster than the space it occupies increases; and a formula expressing the law of acceleration, as depending on all these causes, more especially on the latter, might be expected to be very complex. Nevertheless, it appears from Dr HUTTON's experiments, that the velocity with which a ball actually issues from the mouth of a cannon, other things

things being the same, is nearly as the square root of the weight of the gunpowder, or more generally, if v be the initial velocity of the shot, P the weight of powder, and B the weight of the ball, that $v = m\sqrt{\frac{P}{B}}$; m being a constant co-efficient, to be determined by experiment.

Philosophical Transactions, 1778. Also HUTTON's *Course of Mathematics*, vol. III. p. 270.

Now that $v = m\sqrt{\frac{P}{B}}$, is exactly the conclusion that would be deduced, from supposing the acceleration of the ball to depend simply on the expansion of the elastic fluid, without taking into account the diminution of the impulse, arising from the velocity which the ball has already acquired. This may be shewn from the *Principles of Dynamics*, § 96. That diminution, therefore, does not affect the ratios of the velocities of shot due to different charges, though it must no doubt affect the absolute quantities, and render them less than they would be otherwise.

The value of v given above, is only exact on the supposition, that the piece is long enough to allow the elastic fluid generated by the gunpowder to produce its full effect. The augmentation of the charge may so much lessen the space over which the fluid is to act, that the velocity of the ball shall

shall be less from a great charge than from a smaller.

358. The Balistic Pendulum is an instrument invented by Mr Robins, for determining the velocity with which balls are projected by ordnance of different kinds. The ball is made to strike a heavy block of wood, suspended from a centre; the velocity is thus reduced to a quantity that is moderate, and easily admitting of being measured. From thence the original velocity is computed by help of the principles explained, Mechanics, Sect. 7. on the Rotation of Bodies, § 211.

For a particular description of this instrument, see Robins' *New Principles of Gunnery*, p. 84. Also Euler's *Notes* on Robins' *Gunnery*. The rule for computing the velocity of the ball from the vibration of the pendulum is there investigated, as it is likewise, Hutton's *Tracts*, p. 115., &c. The initial velocity of shot estimated in this way, varies from 1600 to 2000 feet in a second. The latter is nearly as great as it is ever found.

The velocity with which the elastic fluid, disengaged from the gunpowder, expands itself, is probably more than double of this. Dr Hutton estimates it at 5000 feet *per* second. *Course of Mathematics*, vol. III. p. 270.

359. The

359. The depth to which a ball penetrates into wood, earth, &c. is nearly as its weight multiplied into the square of its velocity.

Hence, since $v^2 = m^2 \times \frac{P}{B}$, $B v^2 = m^2 P$; so that the effect of a shot is nearly as the quantity of gunpowder.

From the Table given by Dr HUTTON, *Course of Mathematics*, vol. III. p. 272. the value of m may be deduced nearly $= 2000$.

Bodies impelled by Currents in the Atmosphere, or by Winds.

360. The impulse of a stream of air striking with the velocity v, on a plane of the area a^2, inclined at an angle i to the direction of the stream, $= m \times a^2 v^2 \sin i^2$.

m is a quantity to be determined from experiment, and is constant, while the density of the fluid continues the same.

This proposition is the same that was formerly enunciated of non-elastic fluids, § 301.

361. The sails of a windmill are so disposed as to turn in a vertical plane round a horizontal axis, they themselves being all inclined to that plane toward the same side; when, therefore, the plane in which the sails turn, is placed at right angles to the direction of the wind, the force on each sail is resolved into two, one at right angles to its surface, and the other parallel to it. The latter has no effect, but a part of the former impels the sail, on the side toward which it makes an acute angle with the direction of the wind. Thus a circular or rotatory motion is given to the sails.

The sails are so constructed, as to have different inclinations to the plane of their motion at different distances from the axis, greatest nearer the centre, and least at their extremities. This is called the *weathering* of the sails, and is done in order to make the momentum of the wind nearly the same at all different distances from the centre of motion.

362. Supposing the sail of a windmill to be a plane, and the inclination of that plane to the axis or the direction of the wind to be i, the effect of the wind to turn the sail, in a plane at right angles to its axis, will be the greatest when $\cos i \times \sin i^2$ is a maximum, or $\cos i = \frac{1}{3}$.

This gives $i = 54° 44'$, and therefore the inclination of the sail to the plane of its motion, or what is called

called the angle of *weather*, 35° 16′. This is true only when the sail is at rest, or just beginning to move. When the sail is in motion, and of course near the extremities of the sail, when it moves faster, the angle of weather must be less. MACLAURIN has given a formula for this, *Fluxions*, vol. II. § 913, 914.

MACLAURIN's theorem makes the *weather* to vary from 26° 34, at the point of the sail nearest the centre, to 9° at its extremity. SMEATON made some corrections on this, from experiment. See *Experimental Inquiry* concerning *Mills*, p. 45, and 46.

363. From SMEATON's experiments it appears, that a windmill works to the greatest advantage, when it is so constructed that the velocity of the sails, is to their velocity when they go round without any load, as a number between 6 and 7 is to 10; and also that the load, when the mill works in this manner, is to the load that would just keep it from moving, nearly as 8.5 to 10.

Experimental Inquiry, p. 49 & 50.

364. With different velocities of wind, the load that gives the maximum *effect*, varies nearly as

the square of the velocity of the wind, and the *effect* itself as the cube.

Ibid. p. 52.

The *effect* is always measured by the product of the velocity of the load into its weight. The velocity of the load varies in the simple and direct ratio of the velocity of the wind.

Resistance of the Air to Projectiles.

365. Though we should be led, as in hydraulics, to conclude that the resistance which air makes to moving bodies is as the square of their velocities, experiment appears to prove, especially when the velocity is great, that the resistance is partly proportional to the square, and partly to the simple power of the velocity.

The resistance to the same ball, its velocity being v, is $m v^2 + n v$; where m and n are given co-efficients. HUTTON'S *Course of Mathematics*, vol. III. p. 278.

In this formula $m = .00002576 = \frac{1}{38825}$, and $n = -.00388 = -\frac{1}{258}$, the diameter of the ball being 1.965 inches.

Hence

Hence for any ball of which the diameter is d, $m = .00000666$, and $n = -.001$, that is,

$$m = \frac{2}{3000000}, \text{ and } n = -\frac{1}{1000};$$

and the resistance in avoirdupois pounds, or

$$r = \frac{d^2}{1000}\left(\frac{2v^2}{3000} - v\right).$$

For this theorem, so useful in gunnery, and so well accommodated to practice, we are indebted to Dr Hutton of Woolwich, *ubi supra.*

A ball is often resisted by a force that is many times its own weight. An iron ball 3 lbs. weight has its diameter $= 2.78$ inches, and when it is thrown with a velocity of 1800 feet, it is resisted by a force equal to 176 lbs., more than 58 times its own weight.

366. Supposing the air to resist according to the law just assigned; the height, to which a ball of the weight w, and the diameter d, will ascend, when projected perpendicularly upwards with any velocity c, will be,

$$\frac{w}{2gd^2m} \times \text{hyp. log.} \frac{c^2 - \frac{n}{m}c + \frac{w}{md^2}}{\frac{w}{md^2}}.$$

If

If this expression be reduced, by substituting for w its value $.5236\,d^3$, and using the common logarithms, $h = 760\,d \times \log\left(\frac{c\quad 150\,c}{21090\,d} + 1\right)$.

It may be shewn by this, that a ball 1.05 lb., discharged with a velocity of 2000 feet, will ascend only to the height of 2920 feet, or little more than half a mile, whereas *in vacuo* it would have ascended to the height of eleven miles and three-fourths. HUTTON, *ibid.* p. 284.

367. If a body descending in the atmosphere, has acquired such a velocity, that the resistance is equal to its weight, the accelerating and retarding forces being equal, its motion will become uniform.

Hence, since $.5236\,d^3$ is the weight of an iron ball of the diameter d, if we make

$.5236\,d^3 = \frac{d^2}{1000}\left(\frac{2\,v^2}{3000} - v\right)$, (see article 7.), or

$523.6\,d = \frac{2\,v^2}{3000} - v$, we have a quadratic equation,

from the solution of which v may be found. The velocity thus found is called the *terminal velocity* of the falling body, or of the projectile. For an iron ball of 1 lb., the terminal velocity is 244 feet; for one of 42 lb., it is 456. *Vid.* Table, HUTTON'S *Course*, vol. III. p. 291.

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In strictness this velocity is not acquired till after an infinite time, and a descent infinitely long: the time of acquiring a velocity differing from it, by any quantity, however small, is finite, and can easily be assigned.

368. The great problem in gunnery, viz. having given the weight, the magnitude, the direction, and the velocity of a projectile, to determine its path through the air, supposing the law of resistance to be known, is very difficult, and the solutions of it hitherto given have not led to results easily applicable to practice.

An approximation has been given by NEWTON, and complete solutions by EULER and LE GENDRE. *Prin. Math.* lib. II. Prop. 10. EULER's *Remarks* on ROBINS, BROWN's translation, p. 322., &c. LE GENDRE's solution is in FRANCŒUR's *Mechanique*, p. 196., &c. 3d edit.

Dr HUTTON of Woolwich is in possession of many valuable materials relative to this problem, furnished by his own experiments, and there is reason to expect a more useful solution than has yet been given, from one who unites profound mathematical knowledge with great practical skill.

D'ANTONI has proposed a method of determining by experiment an indefinite number of points in the path of the same projectile.

If a series of stations, one above another, be taken on the declivity of a hill, at the bottom of which there

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is stretched out a plain of considerable extent; and if the same piece of artillery be carried successively to these stations, and fired at the same angle, and in the same circumstances in all respects, and the points marked where the shot in all these cases strikes the plain, it is evident, that all these are points in the curve described by the same shot.

The same might be done still more easily, by removing the gun from one distance to another in the plain, and firing against the side of the hill. If the points from which the shot were fired, and those which they struck, were all marked on an accurate profile of the ground, they would enable us to determine as many points in the path of the projectile as there were experiments made. It is unnecessary to observe, that at each station a great number of shot might be fired at different elevations, so that many curves might be determined from the same set of experiments; and that at each particular elevation many shots ought to be fired, and a mean taken among the points where the balls struck.

369. The ranges of the same ball, with the same elevation, but different charges, are nearly as the square-roots of the initial velocities; and the times of flight are nearly as the ranges.

HUTTON'S

HUTTON's *Mathematical Dictionary*, article *Gunnery*, p. 567. Both these propositions are *empirical*, or deduced solely from experiment.

Beside the works already quoted, see the article *Resistance*, by Professor ROBISON, in the *Encyclopædia Britannica.*

SECT. II.

AIR AS THE VEHICLE OF SOUND.

Vibration of Sonorous Bodies.

370. The bodies that we consider as *sonorous*, or as causes of the sensation of sound, at the time when they produce that sensation, are observed to be in a state of tremor or vibration.

This is exemplified in a bell, and in a drinking-glass when sound is produced by rubbing its edge with a wet finger, in a musical string, &c. The latter affords the simplest case of this fact, and that in which the laws of the vibration are most easily investigated.

371. If a musical string stretched between two fixed points, be struck any where between those points,

points, so as to be forced out of the straight line by a small quantity, it will vibrate backwards and forwards on each side of that line, and the curves into which it will successively pass in the course of these vibrations, will have their curvature at every point proportional to the distance from the straight line joining the fixed points; the accelerating force at each point will also be proportional to that distance, and the great and small vibrations will be performed in the same time.

It is usual to reckon the vibrations of a string differently from those of a pendulum; the passage from the highest point on one side, to the highest on the other, is reckoned *a vibration* of a pendulum. The passage from the farthest distance on one side to the farthest on the other, and back again to its first position, is accounted a *vibration* of a musical string. It is properly a double vibration.

372. The figures which a musical string assumes in its vibrations, constitute a series of elastic curves, or elongated cycloids.

The construction of these curves is easy. See the Notes in Le Seur, and Jacquer's Commentary on the *Principia*, lib. II. prop. XLII. the Note 301. Also Smith's *Harmonics*.

373. If

373. If L be the length of a musical string of uniform thickness, P its weight, S the weight by which it is stretched, or the measure of its tension, t the time of a double vibration,

$$t = 2\sqrt{\frac{LP}{gS}}.$$

t, as usual, is measured in seconds, and g is the velocity acquired in 1″ by a falling body, expressed in the same measure with L. The most convenient unit in this case is an inch; so that $g = 386$.

The value of t may be made more convenient, by supposing w to denote the weight of an inch of the string; so that $P = wL$, then $t = \frac{2L\sqrt{w}}{\sqrt{gS}}$.

If N be the number of vibrations in a given time, for instance in one second, $N = \frac{\sqrt{gS}}{2L\sqrt{w}}$.

If the lengths and weights of two chords are the same, their times of vibration will be inversely as the square roots of the forces by which they are stretched; and the number of vibrations which they perform in the same time, directly as those square roots.

So also, the tension and the weight remaining the same, the celerity of the vibration is inversely as the square root of the length; or, the tension and weight

weight *per* inch remaining the same, the celerity of vibration is inversely as the length.

The problem of the musical chord was first resolved, and the preceding theorems investigated, by BROOK TAYLOR, in his *Methodus incrementorum*, prop. 21. 23., &c. His solution, though very ingenious, and extending to most of the cases that occur in nature, was not general, nor complete, when mathematically considered. The first complete solution was given by D'ALEMBERT, in the *Berlin Memoirs* for 1747, and led him at the same time to discoveries which form a great era in the history of the Differential and Integral Calculus. *Mém. de Berlin*, 1747, p. 214., &c.

374. When the sounds of different musical strings are compared, a certain difference between them is perceived by the ear, which is called *difference of tone;* and this difference is also expressed by saying, that the one sound is *graver*, and the other more *acute;* the variations of tone are found to have a constant or fixed relation to the comparative celerities of vibration.

As the string performs more vibrations in a given time, the sound it yields becomes more acute; and as it vibrates more slowly, the sound is graver. This is easily brought to the test of experiment. The strings that vibrate faster, either from the greater tension, or their smaller length and weight, invariably

invariably produce sounds that are more acute, &c.

375. If eight strings be such, that the numbers of the vibrations they perform in a given time are as the numbers 24, 27, 30, 32, 36, 40, 45, 48, the sounds of the first seven will be perceived as increasing in acuteness one above another, from the first to the last, and will yield the notes from the combinations of which all musical effects are produced.

The tone is not affected by the extent of the vibrations, but merely by their time. The loudness of the sound is supposed to depend on the greater extent of the vibrations. Noise and discordant sounds arise from a want of isochronism of vibration. When the vibrations are isochronous, or all performed in the same time, the sounds are always musical.

The last of the strings will sound what is called the *octave* above the first, and the same series may be repeated again between the number 48 and its double 96, and each note will be an octave above its corresponding note in the first interval; the numbers of vibrations will be 54, 60, 64, 72, 80, 90, 96; and it is evident that this series may be continued either up or down without limit.

The musical scale thus formed, is called the *Diatonic Scale.*

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The pleasure derived from the successive or simultaneous perception of the sounds of this series, appears to be an ultimate fact that can be no farther analysed, but must be referred to the original constitution of the mind. The selection of the combinations of these notes, capable of affording high degrees of pleasure, is the object of the musical art.

377. All other sonorous bodies, at the time they emit sound, vibrate in like manner, but according to laws less simple. In wind instruments, the sounding and vibrating body is the air itself.

The number of vibrations performed in a given time, by any sonorous body, may be determined by comparing its sound with the note which is sounded by a musical string of a given length, weight and tension. The ear is sufficient to decide what string in a harpsichord is in unison with the given sound. The number of vibrations performed in a given time by the former, is equal to the number of those performed in the same time by the latter.

378. All musical sounds are computed to be contained within ten octaves; so that the number of vibrations in a given time that yields the gravest note, is to that which yields the most acute, as 1 to 2^{10}, or as 1 to 1024.

Propagation

Propagation of Vibrations through the Air.

379. The vibrations of sonorous bodies are communicated to the air, and by the impression thus made on the ear, excite the sensation of sound.

That air is necessary to the production of sound, is evident from including a bell in a receiver, exhausting the air, and making the clapper strike on the bell: the sound is hardly audible.

380. It is not every kind of vibratory motion produced in the air that is the cause of sound; a musical string may vibrate, but if it is touched by a bit of cloth, or any soft body, no sound is heard. The vibrations in the air that produce sound must be communicated by some elastic substance.

Sound is produced by the explosion of gunpowder, that is, by the sudden extrication of a fluid mass, dense, and highly elastic. It is produced also by the sudden rushing in of air to supply a vacuum. The crack of a whip appears to be an example of this last.

381. Though the vibrations of the air that produce the sensation of sound, are no doubt the alternate

ternate condensations and rarefactions of that fluid, in consequence of which the particles go and return, or oscillate backwards and forwards for some time, even though there is no renewal of the impulse of the sonorous body, yet every such oscillation acts only once, or by a single impulse on the ear. Were it otherwise, sound would be always something inarticulate and ill defined.

382. The velocity with which vibrations are propagated through the air, is the same that a heavy body would acquire by falling through half the height of the *homogeneous* atmosphere, or that which the atmosphere would be reduced to, if it were everywhere of the same density, and the same temperature with the air at the surface of the earth.

The height of this homogeneous atmosphere has been formerly computed at 4343 fathoms, when the temperature is that of freezing. If this height be called H, then v, the velocity of the aerial vibrations, $= \sqrt{2gH}$. Hence $v = 1057$, which is too small, as it is found by experiment to be 1142 feet *per* second.

The nature of the vibrations of an elastic fluid, was first considered by NEWTON, and their velocity defined, as in the preceding proposition, by comparing them with the vibrations of a pendulum. *Princip.*

Princip. Math. lib. ii. prop. 47. & 49. The soundness of this reasoning, however, was questioned by EULER; and it was afterwards shewn by CRAMER, that the same argument might be used to prove conclusions very different from that at which NEWTON had actually arrived. *Commentary on the Principia*, lib. ii. vol. ii. p. 364. LA GRANGE afterwards gave an accurate solution of the problem, by which, though he pointed out the error of NEWTON's reasoning, he confirmed the truth of his conclusion.

383. The velocity of the pulses propagated in an elastic fluid, are as the square root of the elasticity divided by the density of the fluid.

Prin. Math. ibid. prop. 48.

The velocity of sound, computed as in the last article, comes out less than by actual experiment. LA PLACE has suggested a very probable explanation of this, viz. that the condensation of the *undulæ*, which must take place in these vibrations, produces a degree of sensible heat, by which the elasticity is increased, or, to speak more correctly, the density diminished, while the elasticity remains the same. The heat required for that effect has been shewn by BIOT to be within the probable limits which analogy, in the absence of direct experiment, would lead us to assign. HAÜY, *Leçons de Phys.* § 478.

 384. There

384. There is perhaps, in the economy of nature, no contrivance more wonderful, than that by which things apparently so little susceptible of precision as the impulses communicated to an elastic fluid, become the means of conveying to the mind such a multitude of distinct impressions as it receives through the ear; the finest modulations of harmony, and the nicest distinctions of articulate language.

385. An Echo is a repetition of sounds produced by the reflection of the aërial pulses that convey sound to the ear.

A wall, a rock, a grove of trees, may be so placed, as to cause an echo. That an echo may return one syllable as soon as it is pronounced, the reflecting surface should be 80 or 90 feet distant; for a dissyllabic echo, 170 feet, &c. The sound, whether direct or reflected, is supposed to proceed at the rate of 1142 feet in a second.

386. The Speaking Trumpet is an instrument intended to transmit the sound of the voice in some particular direction, to a greater distance than it would otherwise reach.

The best form of a speaking-trumpet, is found to be that of a hollow cone, with a mouth-piece at the narrow

narrow end, to receive the lips, and confine the voice of the speaker.

The beat of a watch may be heard to twice the distance through a speaking-trumpet, that it can be heard at without one. It is said, to be the same in this experiment whether the trumpet is cylindrical or conical. See the *Edinburgh Encyclopædia*, article *Acoustics*, Part ii. Sect. ii. This instrument seems to produce its effect, by preventing the pulses, as they are generated, from diffusing themselves all round, and subjecting them longer to an impulse in one direction.

Sect. III.

AIR, AS THE VEHICLE OF HEAT AND MOISTURE.

387. The Earth and the Atmosphere, taken generally, receive at all times nearly the same quantity of heat and light from the sun.

As the whole of one side of the earth is constantly turned to the sun, the only difference in the quantity of heat and light which the earth receives in a given time, must arise from the changes which take place in its distance from the sun at different seasons of the year. In consequence of these changes, the quantities of light and heat received by the earth are not proportional to the times, but to the angles described by the earth round the sun in those times. These variations, however, are but inconsiderable, and as they are annual, they do not produce any inequality in the whole heat or light of one year compared with those of another.

388. Though the earth is thus receiving heat continually, and nearly at the same rate, its average temperature appears to remain invariable; as much heat as comes from the solar rays, flying off continually into the fields of vacuity, or of ether.

389. While the general temperature of the earth remains invariable, the distribution of heat over its surface is extremely unequal, being different in different places, and in the same place subject to variations, both regular and irregular.

390. The causes that determine the distribution of heat over the earth's surface, are either the direct influence of the solar rays, or the communication of heat by the air from one part of the earth's surface to another.

391. The first of these depends on the latitude of the place, or its distance from the equator, by which the intensity of the heat and light from the sun, and also the length of the day, are determined for different seasons of the year.

The intensity of the sun's rays, when they strike on any plane, is as the quantity that falls on a given space, or as the sine of the sun's elevation above the plane. The nearer the sun is to the zenith of any place, at a given moment, the greater the intensity of heat produced by his rays.

The heat for an entire day, depends also on the length of the day, and as the day is longer where the distance from the zenith is greater, the inequality in the distribution of heat arising from the one of these causes, compensates that proceeding from the other, and brings their combined effects nearer to an equality than might be imagined. FONTANA has shewn, that the heat of the day of the summer solstice at Pavia, is greater than the heat of the same day at Petersburgh only in the ratio of 63° to 62°, though the latitude of the former be 45° 11′, and of the latter 59° 56′.

The same author finds, that when the sun's declination exceeds 18°, or from about the 10th of May to the 30th of July, the heat in twenty-four hours proceeding from the sun's rays is greater at the north pole than at the equator. GREG. FONTANA, *Disquisitiones Physico-Mathematicæ*, 1ma & 2da.

The

The distribution of heat, therefore, if only the direct influence of the sun were to act, would be very different from that which takes place in nature.

The intensity of the solar heat is less in low elevations than is here supposed, on account of the rays coming through a larger mass of air, and of air more loaded with vapour, so that a great quantity of them is intercepted. BOUQUER, *Traité d'Optique; sur la Gradation de la Lumiere,* Liv. iii. sect. iv.

392. The effects of the direct influence of the sun, are greatly modified by the transportation of the temperature of one region into another, in consequence of that disturbance in the equilibrium of the atmosphere which the action of those rays necessarily produces.

In order that there may be an equilibrium in a fluid like the air, every stratum of air that is level or horizontal all round, ought to be of the same density. It ought, therefore, also to be everywhere of the same temperature, which not being the case, the constant motion of the air is the necessary consequence of heat being unequally distributed. The columns of air that are lighter, are displaced by those that are heavier, and hence a general tendency in the air to move from the poles toward the equator. This general tendency, which is calculated to moderate the extremes of temperature, is also greatly modified by local circumstances.

393. The

393. The sea is preserved of a moderate temperature by the statical principle that makes the heavier columns of a fluid displace the lighter. A more uniform temperature is thus given to the sea, which communicates itself to the air incumbent on it, and to that on every side.

394. Conversely, the effect of great and unbroken continents, is favourable to the extremes of heat or of cold.

The constitution of the surface may tend to increase and sometimes to diminish this effect. High mountains especially, if covered with snow, may enforce the rigour of a cold climate, or temper the heats of a warm one.

Forests tend to increase the cold, by preventing the sun's rays from striking on the ground. Evaporation produces cold; and marshes and lakes are therefore favourable to the severity of the weather. The congelation of water produces heat, and moderates the cold; the melting of ice, on the other hand, increases the capacity for heat, and so produces cold.

395. Height above the level of the sea, causes a diminution of heat at the constant rate of 1° for 270 feet nearly, when not far from the surface of the earth.

It has already been remarked, that this decrease seems to be somewhat slower as we ascend, but not very considerably, as far as our observations have extended.

396. The combination of these causes gives to every place a mean temperature, which remains always nearly the same, and which decreases from the equator to either pole, according to a law that has been determined by observation.

397. Let t be the mean temperature of any parallel of which the latitude is L, M the mean temperature of the parallel of 45°, and M + E the mean temperature of the equator then is ;

$$t = M + E \cos 2L.$$

In this formula, M = 58°, and E = 27°. When $2L > 90$, $\cos 2L$ is negative.

This theorem was first given by MAYER, *Opera inedita*, vol. i. p. 4., &c. Also KIRWAN, *Estimate of the temperature of different Latitudes*, p. 18.

From this a geometrical construction, for finding the mean temperature, may be readily deduced.

In the line AC (fig. 25.), divided into equal parts, numbered from A, so as to represent the scale of a thermometer, let AC = 85, and AB = 58. From the centre

centre B, with the distance BC or 27, describe the semicircle CA; take the arch CG equal to the double latitude of any parallel; and from G draw GO perpendicular to AC; then is AO the mean temperature of that parallel, according to FAHRENHEIT's scale.

The mean temperatures thus found, agree very well with observation. Springs, in which the water does not considerably change its heat from one season of the year to another, afford an expeditious and accurate way of ascertaining the mean temperature.

398. If the place is at any height H above the level of the sea, $t = M - \frac{H}{270} + E \cos 2L$.

H is understood to be expressed in English feet.

399. On ascending into the atmosphere, at a certain height in every latitude, a point is found where it always freezes, or where it freezes more than it thaws, so that the mean temperature is below 32°. The curve joining all these points, from the equator to the pole, is called the *line of perpetual congelation.* The equation to it will be found, by making $32 = M - \frac{H}{n} + E \cos 2L$.

This

This line at the equator is elevated 15577 feet above the level of the sea; so that for determining n we have $32 = 58 - \frac{15577}{n} + 27$, or $n = 294$.

Thus, $H = 7642 + 7933 \cos 2L$, and this seems nearly to agree with actual observation.

Professor LESLIE has given a different equation, founded on the law of the diminution of heat mentioned at § 339. In the table calculated from it, heights come out rather below what observation requires. *Elements of Geometry*, 2d edition, p. 495.

400. The temperature of the latter end of April is observed, at least in the temperate zone, to be nearly the mean temperature of the year. From that time the heat increases, and is at its maximum about the 21st of July; it goes on decreasing from that time till it come to the mean in the end of October, and it passes from thence to the greatest cold about the 21st of January. All these vicissitudes may be nearly represented by the formula in which G is the mean temperature for the given place, F a constant co-efficient to be found by observation, λ the mean longitude of the sun computed from the first of Aries, for any day of the year, the mean temperature of which is y. Thus, the times of the mean and the extremes of temperature being taken by about one month later

ter than the corresponding mean and extremes of the sun's elevation,

$$y = G + F \sin(\lambda - 30).$$

This supposes that the mean heat takes place when the sun enters Taurus, which is nearly true. In this latitude, we may suppose $F = 15°$.

401. From this formula, compared with the two former, one theorem may be deduced, including the effects of latitude, elevation above the surface, and the season of the year, viz., the mean temperature of any day under any parallel, and with any elevation,

$$y = M - \frac{H}{270} + E \cos 2L + F \sin(\lambda - 30°).$$

402. The formula for the mean temperature, as laid down above, would probably hold over the whole globe, if it were everywhere covered by the ocean; but it agrees strictly only with the Atlantic Ocean, and the western part of the Old Continent.

The editor of MAYER's Posthumous Works observed, that his theorem applied accurately only to the portion

portion of the globe contained between the parallels of Stockholm and of the Cape of Good Hope, and between the meridians of Stockholm and Mexico. This does not seem quite exact; no part of North America having its mean temperature the same with that of places of the same latitude in Europe. It would require at least 10° to be taken from M, to adapt the formula of § 397. to the New Continent. See KIRWAN's *Estimate of the temperature of different Latitudes*, p. 15.

403. As we go eastward from the shores of the Atlantic, the mean temperature of any parallel becomes lower, at a rate that may perhaps, for the north part of the temperate zone, be estimated at a degree for 150 miles.

At St Petersburgh, lat. 59° 56′, about 750 miles from what may be accounted the shores of the Atlantic, the temperature is 5° 5′ below the standard. The medium temperature of January is no more than 10°. By computation from the formula above, it ought to be greater than 32°. The winter lasts from October to April, and the cold is sometimes as great as the freezing point of mercury, or —39°. From a mean of several years, the mean of the winter cold is —25°. KIRWAN, *ibid.* p. 61.

It was at Krasnojark, lat. 56° 30′, long. 93° E. that mercury was first known to freeze by natural cold.

If

If we were to begin where any parallel intersects the shore of the Atlantic, and draw on the map a line along which the mean temperature should be constantly the same as at the first-mentioned point, it would incline greatly to the south. The point, for instance, in the meridian of Petersburgh, which has the same temperature with the standard belonging to the parallel of that city, is about 5° south of it, or in the latitude of 54° 30′ nearly.

At Irkutz, latitude 52° 15′, longitude 105° east, the mean temperature from October to April has been known to be as low as —6°.8, a temperature which for severity and duration exceeds any thing that has yet been observed elsewhere.

404. This increase of the severity of the winter, and the consequent diminution of the mean temperature, on going eastward, holds in all the latitudes north of the parallel of 30°; but the diminution is slower as we approach that parallel; to the south of 30° the mean heat increases on retiring from the ocean.

This diminution takes place all the way to the shores of the Pacific, or very near them. The climate of Pekin is vastly more severe than that of the same parallel (39° 54′) in Europe.

405. In the New Continent, also, at least in the part of it to the north of the Tropic of Cancer,

the mean temperature is much below the standard, and the severity of the winter much greater than in the same latitudes in Europe.

At Prince of Wales' Fort, Hudson's Bay, lat. 59°, long. 92° west, the mean temperature is 20° under the standard; at Nain in Labrador 16°; at Cambridge in New England (lat. 42° 25',) 10 degrees. Mercury has been supposed to be frozen by the natural cold as far south as Quebec, lat. 47°.

A very low mean temperature, and extreme cold in winter, are characteristic of the climate of North America.

406. In the higher latitudes of the Southern Hemisphere, the temperature is lower than in the same latitudes of the Northern Hemisphere.

Forster describes a small island on the coast of South Georgia, latitude 54° south, which, in the middle of summer, was covered almost entirely with frozen snow, to the depth of several fathoms.

The South Pole is surrounded to the distance of 18 or 19 degrees with a barrier of solid ice, through which even the skill and intrepidity of Captain Cook could not force a passage.

It is known also, that detached masses of ice float down in that hemisphere as low as the latitude

tude of 46°. The cause of this phenomenon is by no means sufficiently understood.

407. The varieties of temperature to be met with on the surface of the earth, appear to be confined between the limits of 100° and —40°.

No degree of cold much below —40° has ever been observed in nature, even when thermometers have been employed containing a fluid not liable to congelation. Sir CHARLES BLAGDEN *on the Congelation of Mercury*, p. 61.

The heat of 100° is rare, but a heat approaching to 90°, is found in the summer of most countries within the limits of the temperate zones.

There is hardly any climate, even in the frigid zone, where a temperature between 60° and 50° is not occasionally experienced.

The greatest heat is much less above the mean temperature than the greatest cold is below it. The mean temperature for the whole surface may be taken at 58°; the greatest summer heat is only 42° above this; the greatest winter cold in 98° under it.

408. There is reason to think that the climates of Europe were more severe in ancient times than they are at present, and the change that has taken place may with great probability be ascribed to

the better and more extensive cultivation of the ground.

CÆSAR says, that the vine could not be cultivated in Gaul on account of the severity of the winter. The rein-deer was then an inhabitant of the Pyrenees. The Tiber was sometimes frozen over, and the ground about Rome covered with snow for several weeks together. See HUME *on the Populousness of Ancient Nations*, *Essays*, vol. i. p. 451. edit. 1772. DAINES BARRINGTON, *Phil. Trans.* vol. lviii. (1768) p. 58., &c.

Cultivation may improve a climate, 1mo, By the draining of marshes, and lessening the evaporation, which is so great a cause of cold; 2do, By turning up the soil, and exposing it to the rays of the sun; 3tio, By thinning or cutting down forests, which by their shade, prevented the penetration of the sun's rays. The improvement which is continually taking place in the climate of North America, proves that the power of man extends to the phenomena, which, from the magnitude and variety f their causes, seemed most superior to his controul. At Guiana in South America, the rainy season has been shortened by the clearing of the country, and the warmth greatly increased. It thunders continually in the woods; rarely in the cultivated parts. BUFFON, *Sup.* tom. ix. p. 346. octavo.

Wind.

Wind.

409. The principal cause of those currents of air to which we give the name of *Winds,* is the disturbance of the equilibrium of the atmosphere by the unequal distribution of heat.

In order that an equilibrium may take place in an elastic fluid, circumfused about a solid, to which it gravitates, every level stratum of the fluid, that is, every stratum, which, when continued round, cuts the directions of gravity everywhere at right angles, should be of the same density, and therefore of the same temperature. As this is not the case, the equilibrium of the atmosphere is inconsistent with the actual distribution of heat on the earth's surface.

The general tendency, in such circumstances, is for the heavier columns to displace the lighter, and for the air at the surface to move from the Poles toward the Equator. The only supply for the air thus constantly abstracted from the higher latitudes, must be produced by a counter-current in the upper regions of the atmosphere, carrying back the air from the Equator toward the Poles. The quantity of air transported by these opposite currents, is so nearly equal, that the average weight of the air, as measured by the barometer, is the same in all places of the earth.

If the surface of the earth were wholly covered with water, so that there was no part of it more disposed than another to obstruct the motion of the air, or that had a greater capacity than another, of acquiring or communicating heat, the air would probably circulate continually in this manner from the Poles to the Equator, and back again, without any irregularity whatsoever.

410. In consequence of the rotation of the earth on its axis, another motion is combined with that of the currents just described. The air, which is constantly moving from points where the earth's motion on its axis is slower, to those where it is quicker, cannot have precisely the same motion eastward with the part of the surface over which it is passing, and therefore must, relatively to that surface, describe a curve, having its convexity turned to the east. The two currents, therefore, from the opposite hemispheres, when they meet toward the middle of the earth, have each acquired an apparent motion westward; and as their opposite motions from south and north must destroy one another, nothing will remain but this motion, by which they will go on together, and form a wind blowing directly from the east.

This is the cause of the *Trade Wind*, which (with certain exceptions) blows continually between the Tropics,

Tropics, or rather between 30° on the one side of the Equator, and 30° on the other.

The Trade Wind declines somewhat from due east toward the parallel to which the sun is vertical at different seasons of the year. As the sun approaches the southern tropic, the Trade Wind is directed somewhat to the south; and as he approaches the northern, somewhat to the north.

The cause usually assigned for the Trade Wind, is the constant motion toward the west of the spot to which the sun is vertical, and where of course the rarefaction is greatest. This, it is supposed, draws along with it the air from the east. This, however, is by no means satisfactory, and it seems certain, that if the Trade Wind were produced in this way, it must have great rapidity, in place of being a gentle breeze, at the rate of seven or eight miles an hour.

The opinion that the Trade Wind is produced by the air in its motion southward falling back toward the west, is mentioned, but rejected, by Halley. It has since been espoused by Franklin and La Place, and is, on the whole, less objectionable than any other.

The matter is here stated somewhat differently from what is done by those authors, particularly the effect of the currents from the opposite hemispheres, in determining the motion to be wholly from the east.

411. The attractions of the sun and moon have sometimes been considered as among the general causes of the winds. They have no doubt a tendency to produce in the atmosphere an undulation backwards and forwards, like the tides which they cause in the ocean. It does not appear, however, that they could produce any continued progressive motion of the air, similar to that of the Trade Winds. Their effects also, are too minute to be perceived, amid the action of so many more powerful causes.

D'ALEMBERT in his *Recherches sur les Causes Générales des Vents*, has treated of the forces of the sun and moon to produce currents in the atmosphere. His essay is more remarkable for the resource and ingenuity it displays in the management of the *calculus*, than for the physical conclusions to which it leads.

412. The superior current above described, restores the air carried from the higher latitudes to the lower, with such a degree of equality, that the average weight of the atmosphere, as measured by the barometer, is nearly the same in all climates. This restoration is, however, subject to great local and temporary irregularities, from the different degrees

degrees of resistance that the air meets with in passing over the surface, and the different capacities of that surface for receiving and communicating heat.

The motion of the inferior and superior currents, may be seen exemplified on opening a door, between two apartments of different temperatures. The flame of a candle near the ground, will shew the stream that sets from the colder room to the warmer; near the top it will indicate a stream in the opposite direction.

As the average quantities of the air carried by these opposite currents are equal, the surface that separates them is probably not far different from that at which the barometer would stand at 15 inches, or half its medium height at the surface.

If we suppose the mean temperature is 32°, this elevation will be found = 3010 fathoms, or 18,060 feet = 3.425 miles, not so high as the summits of the Cordilleras.

The upper stream in each hemisphere being directed to one point, the pole of that hemisphere, there must arise a considerable condensation, and acceleration of the air, as the currents approach that point, and hence the causes of irregular winds must be increased on approaching the poles.

The general direction of the upper current must be to the westward, for the same reason that that of the lower was toward the east.

413. The

413. The Trade Wind itself is subject to certain irregularities. As the sun advances into the Northern Hemisphere, the Trade Wind becomes irregular; and about the middle of April, in all the tract between Africa and the peninsula of India, and much beyond it, changes from north-east to south-west, and continues to blow in that direction, till the sun returns to the Southern Hemisphere.

The cause of this change is difficult to be assigned. It seems probable, that by the sun's entry into the Northern Hemisphere, he communicates great heat to the sandy desarts of Africa, which lie to the west or south-west of the seas just mentioned. The great heat acquired by the sand of those desarts, produces a rarefaction in the columns of air incumbent on them, and consequently a tendency, in the columns that are near them, and more moderately heated, to flow in and displace the heated air. The air of the Atlantic is most likely to do this, and in passing over Nigritia, &c., to acquire a velocity that carries it on eastward through the Indian Ocean.

These periodical winds are called *Monsoons*.

The change, or the setting in of the monsoons, does not happen all at once. In some places the shifting of the wind is accompanied with calms; in others with variable winds, heavy rains, thunder, and violent storms.

414. The

414. The tract from the parallel of 30° to the Pole, in each hemisphere, is the region of variable winds; and their unsteadiness and violence seem to increase, the nearer they approach the Polar Circles.

The irregularity of winds proceeds from inequalities in the motion of the general currents above mentioned, and from a variety of local causes; also from the chemical changes that are carried on in the air, such as the solution and precipitation of moisture, and the action of the electric fluid on the different gases that compose the atmosphere.

415. Sudden and strong gales of wind appear almost always to arise from a diminution of the weight of the air in the tract where the wind prevails, and are accompanied, or preceded, by a fall of the barometer.

The sudden sinking of the barometer almost always indicates a gale of wind, though a gale that is sometimes at a considerable distance. When the barometer begins to rise, it is a symptom that the gale has reached its height; and though it may still continue to blow for a long time, it is usually with decreasing violence.

416. Notwithstanding these irregularities, there is in most countries a tendency to periodical winds,

winds, more or less remarkable, according to the steadiness of the climate.

Even with us, where an insular situation, with a great Continent on one side, and a great Ocean on the other, unites all the causes of a variable climate, the East wind usually prevails in the spring, from the vernal equinox to the summer solstice, and beyond it; during the rest of the year, the Westerly winds prevail, though not without frequent incursions of the east, by which our most unpleasant weather is always produced.

The *Etesian*, or northerly wind, prevails very much in summer all over Europe. Pliny describes it as blowing regularly in Italy, for forty days after the summer solstice, lib. ii. cap. 47. It is part of the great current that carries the atmosphere of the higher latitudes down to the tropical regions.

417. The velocity of the wind varies from one that is hardly sensible, to one of 100 miles in an hour.

The force of the wind, is as the density of the air multiplied into the square of its velocity. Smeaton has given a table of its force, expressed by its pressure in avoirdupois pounds, on a plane of one foot square directly opposed to it. *Phil Trans.* vol. li. p. 165., &c. Also Cavallo, vol. ii. p. 287.

A gentle light breeze is there estimated as moving at the rate of four or five miles an hour, and pressing with

with a force of about two ounces: A brisk but pleasant gale from ten to fifteen miles; with a force from half a pound to a pound: A high wind goes at the rate of thirty or thirty-five miles, and presses with the force of five or six pounds: A hurricane, that tears up trees, and blows down houses, has a velocity of one hundred miles an hour, and a force of forty-nine pounds on a square foot.

The velocity of the wind may be estimated by the velocity of clouds, or of light bodies carried by it.

Rain.

418. The vapour that rises from water uniting itself to the air, ascends into the higher regions of the atmosphere, and is carried by the winds to great distances.

It cannot be doubted, that the humidity raised in this manner is chemically dissolved in the air.

The humidity does not lessen, but increases the transparency of the air, and cannot be withdrawn from it but by substances that attract it powerfully.

419. The power of air to dissolve humidity increases in a greater ratio than that in which its temperature increases.

It

It appears from SAUSSURE's experiments, that while the temperature increases in arithmetical progression, the humidity which the air is able to contain increases in geometric progression. A cubic foot of air, of the temperature 66°, is able to hold in solution 11 or 12 grains of water: the air itself weighs 570 grains: so that air of the temperature 66°, dissolves about a 50th part of its own weight of water. *Essai sur l'Hygrometrie*, XI. chap. X. p. 167., &c.

420. Hence, if two portions of air, of different temperatures, and both saturated with humidity, be mixed together, a precipitation of humidity must necessarily take place.

If the abscissæ AB, AB′ (fig. 26.) be the temperatures of two equal portions of air, and the ordinates BC, B′C′ represent the quantities of humidity contained in them, the curve CFC′, which represents the dissolving power, corresponding to different temperatures, will be convex toward AB, and will, so far as experiment has shewn, be a logarithmic curve. Now if CC′ be joined, and if BB′ be bisected in D, the perpendicular DE will represent the humidity contained in a given weight of the mixture of the two portions of air, AD the temperature of the mixture, and DF the quantity of humidity which the air of the temperature AD is able to contain. The quantity precipitated from each portion of the air is therefore EF; so that the whole quantity precipitated is 2 EF.

In

In general, if T and t are the temperatures of two equal portions of air, H and h the humidity contained in them when saturated, the quantity of humidity precipitated from the mixture will be,

$$H + h - 2\sqrt{Hh}.$$

421. If, therefore, large portions of the atmosphere, of different temperatures, and saturated, or nearly saturated, with humidity, be driven against one another by contrary winds, the consequence must be a precipitation of humidity, or the formation of clouds.

This simple and ingenious theory of the formation of clouds, was the discovery of the late Dr HUTTON. See *Edinburgh Transactions*, vol. i. p. 41. It is not necessary to the theory, that the curve which terminates the perpendiculars BC, B′C′, &c. should be a logarithmic curve; it is sufficient that it be a curve convex toward the axis AB. Of this Dr HUTTON had satisfied himself by the observation of natural phenomena. If the line terminating the perpendiculars was straight, and much more were it convex, no precipitation could ever take place.

422. The clouds thus formed, are not disposed equally over the whole atmosphere, but occupy a peculiar region, elevated at an average between two and three miles above the earth.

The

The mixture of different portions of air is likely to take place most frequently when the two opposite currents already mentioned come in contact with one another. This is at the height of 18,000 feet and upwards, which agrees very well with the medium height of the clouds.

423. The clouds thus formed, have their particles united into larger masses or drops by different causes, such as the mutual attraction of aqueous particles, the force of the wind, or the operation of electricity, and so fall down in rain on the surface of the earth.

The intimate connection between rain and the existence of different currents of air, is evident from many appearances.

1. When the Trade Winds blow uniformly, hardly any rain falls; but when the monsoon changes, heavy falls of rain seldom fail to take place.

2. In the tropical climates, the rainy season is always on the sun's approach to the zenith, at which time also the winds are most variable.

3. There are some spots of continual rain, which seem to be where opposite streams of air constantly meet one another.

4. There are several tracts on the earth's surface, where it hardly ever rains. They are usually far inland, and have extensive plains, without any of those

those inequalities of surface that promote the mixture of air.

5. In the midst of those deserts, where mountains occur, moisture is precipitated, sometimes in the form of rain, but most frequently of dew, so that there are springs of fresh water, and great fertility produced.

6. There is in our climate hardly any instance of rain without a change of wind, and very rarely a change of wind without rain in a greater or smaller quantity.

7. The lowness of the mercury in the barometer is a sign of rain. It is a certain indication of the subversion of the equilibrium of the atmosphere, and makes it probable, that before the equilibrium is restored, winds from different quarters, and of different temperatures, must come into collision with one another.

424. The Hygrometer is an instrument intended to measure the quantity of humidity contained in the air at any time. The power of evaporation to produce cold has been very happily applied by Professor LESLIE, to the construction of an instrument of this kind.

425. The vapour raised up into the air, is of a quantity sufficient to afford all the rain that falls,

or to supply all the springs, and of consequence all the rivers derived from them, on the surface of the earth.

Dr Halley shewed, that the evaporation from the sea alone is a sufficient supply for all the waters that the rivers carry into it. His calculation was founded on a very complex view of the subject, and liable to several objections. Buffon took a more simple view of the matter, by selecting one of those lakes that sends out no stream to the ocean, and shewing that the probable evaporation from the surface of the lake is equal to all the water carried into it.

A simpler view still may be taken, by shewing that all the water annually emptied by a river into the sea, is less than the rain that falls on the surface drained by it. Thus, according to Dr Halley's computation, the water which the Thames carries down through Kingston Bridge, to which the tide does not reach, is 25344000 cubic yards, or 684288000 cubic feet *per* day, which gives 249765120000 cubic feet for the quantity of water which the Thames discharges annually into the sea. Now, the surface drained by the Thames and its branches, appears to be about equal to a circle of 40 miles radius; that is, nearly to 5036 square miles, or 140395622400 square feet. But if we suppose that the depth of rain which falls annually, at an average, over this surface is two feet, we have a quantity which exceeds that carried down

down by the Thames by 31026124800, or nearly an eighth part of the whole, which is therefore left to be taken up by the evaporation.

Mariotte has made a similar computation for the Seine, in his *Traité de Mouvement des Eaux*.

426. The disputes that prevailed so long concerning the origin of fountains and rivers, chiefly arose from the difficulty of conceiving, how a precarious and accidental supply could be rendered equal to a regular and great expenditure.

The opinions of the ancients, concerning the origin of fountains, are to be found in Seneca, *Nat. Quæst.* lib. iii. See also Varenius, sect. iv. chap. xvi. prop. 5.

427. The quantity of rain that falls in different places in the same year, and in the same place in different years, is extremely various; and even in the temperate zone runs between the extremes of 18 and 100 inches.

In places not very distant from one another, the difference in the quantity of rain is often very great. The neighbourhood of the sea on one hand, and mountains on the other, is most favourable to the production of rain; from the first is derived a humid

mid atmosphere, and from the second the prevalence of winds of different temperatures.

When the rain exceeds 25 inches a-year, the climate is to be accounted moist. In the year 1809, the rain that fell at Dalkeith, near Edinburgh, was 28.5; at Glasgow 25.1; at Largs, at the mouth of the Clyde, 40.6; and at Gordon Castle, on the east coast of Scotland, 24.5: At London, in the same year, 20.7; in the preceding year, the fall was but 18.8. In some places of this island, such as Kendal and Keswick, the rain amounts to 64 and 68 inches in a year.

428. The disposition of the rocks in strata, contributes much to the collecting of the waters under the surface, and the conveying them without waste, as if in close pipes, till they are united in fountains, lakes, rivers, &c.

Bernard Pallassy was among the first who made this remark, and even brought it to the test of experiment.

429. The region of the air in which the precipitation of humidity takes place, is frequently one where the temperature is below freezing; the frozen particles then uniting, as in the case of melted vapour, form flakes of Snow, which reach the surface in that same state, when the cold of freezing continues down to the surface.

When

When the aqueous particles first form a drop, and are afterwards frozen in their descent, they become Hail, which is sometimes found crystallised with some degree of regularity. The whiteness and opacity of the hail, is probably owing to the congelation being performed where the air is very rare. Professor Leslie has remarked, in the curious experiments he has made on the production of ice by evaporation, in a receiver where the air was considerably rarefied, that the ice is more porous, and less transparent, than that which is formed under the ordinary pressure of the atmosphere.

430. Dew is a precipitation of humidity from the lower strata of the atmosphere, which does not disturb the transparency of the air, and to which the mixture of different streams of air does not seem necessary.

Dew is formed only in calm weather, and in the evening, or during the night. The resting of the globules of dew on the points of grass and other plants, seems to indicate the presence of some electrical influence. Bruce has remarked, that during the dry season no dew falls in Egypt, but that it is usually observed in the Delta five or six days before the inundation.

431. Besides these deposites from the atmosphere, which are all of one substance, there are others

others that cannot be traced to the same origin. These are the Stones which have often been said, and have of late years been ascertained beyond all doubt, to fall down from the air; on examination, they are found to contain certain metallic ores, in many respects unlike to any thing that is found either on the surface or in the bowels of the earth.

See an account of Meteoric Stones, HOWARD, *Phil. Trans.* 1802. IZARN, *Lithologie Atmospherique*, Paris, 8vo, 1803: and CAVALLO, vol. iv. p. 372., &c.

In the absence of all analogous appearances, it is perhaps unphilosophical to offer any explanation of the Meteoric Stones. We would only suggest as a mere possibility, that gaseous substances may be thrown up into the air, from the numerous volcanoes on the earth's surface, which may contain the elements of metallic and stony bodies from the mineral regions; and that these, while floating about in the atmosphere, may sometimes be collected in considerable quantities into one place, where, being subjected to electric or galvanic action, they are united into a solid mass.

END OF VOLUME FIRST.